2014

Prioritization, security and relay selection in network coded multiple access relay networks

Navneet Malani
Iowa State University

Prioritization, security and relay selection in network coded multiple access relay networks

by

Navneet Malani

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Electrical Engineering

Program of Study Committee:

Sang Wu Kim, Major Professor

Zhengdao Wang

Aditya Ramamoorthy

Ahmed Kamal

Yong Guan

Iowa State University

Ames, Iowa

2014

DEDICATION

I would like to dedicate this thesis to my dear family members without whose support I would not have been able to complete this work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENTS

I would like to take this opportunity to express my thanks to those who helped me with various aspects of conducting research and the writing of this report. First and foremost my adviser Dr. Sang Wu Kim for his guidance, patience and support throughout this research and the writing of this report. His insights and words of encouragement have often inspired me and renewed my hopes for completing my graduate education.

I would like to thank Dr. Zhengdao Wang, Dr Aditya Ramamoorthy, Dr Yong Guan and Dr. Ahmed E Kamal for their help and support and their courses which I thoroughly enjoyed.

A special thanks to my parents, my wife, my sister, my brother-in-law, my cousin brother and my sister-in-law for their love and support. Without their constant encouragement, I wouldn't have been able to finish my research. I thank the almighty Shrinathji for giving me strength to complete this work.

My research would haven't been possible without help of my group mates Taha Khalaf, Young Jin Chun and Duk Hee Yoon whom I thank for their help and interesting discussions. I would also like to thank my friends Vinay Jethava, Sunil Srinivasa and Saurabh Ganeriwal for their kind support.

A warm thanks to my all friends and Sankalp executive committee of 2012 - 2013 who made my stay in Ames a memorable one. I thank the staff of Electrical and Computer Engineering department, especially Vicky Thorland-Oster, Sara Harris and Pam Meyers(retd.) in helping me out with all the official work.

ABSTRACT

Wireless communication is undoubtedly one of the most significant advancements by the mankind for improving quality of life. Information is transmitted from one point to another via electromagnetic waves. After Shannon's landmark paper "A Mathematical Theory of Communication" in 1948, significant advancements have occurred in providing reliable point to point wireless communication. With ever growing need for reliable high speed links, Cooperative communication and Network coding have emerged as viable technologies to bridge the gap.

In today's wireless network, different users have different demands for reliability based on their respective application. In this context, we propose flexible network coding scheme to adapt to user needs. We develop coding rules which achieve maximal diversity of the system, yet provide differentiated class of service to the users. The proposed scheme can be adjusted to accommodate the dynamic changes in quality of service(QoS) demand of users. Second we consider the issue of security in multiple access relay network. Security has always been a primary concern in wireless networks due to it broadcast nature of transmission. The intermediate relay nodes in a wireless network could be modified by adversary to transmit corrupted information. We propose a novel iterative packet recycling methodology which gives performance improvement over traditional approach of discarding received corrupted packets at the destination. Finally, we consider the problem of choosing relay for transmission. We propose a novel selection scheme which provides balanced relay utilization and reduces relay switching rate compared to the traditional selection algorithms. This cuts down energy wastage at the relay and improves the overall network lifetime.

CHAPTER 1. INTRODUCTION

In past few decades, there has been surge in wireless technology and wireless users. Since past few decades, there has been constant push to increase capacity, improve reliability and reduce the cost of wireless devices. The demand of mobile telephone service served as a major driver to expand wireless connectivity over few decades which has now given way to wireless data applications. In parallel, the dramatic progress in VLSI technology has made it possible to produce cheap and reliable mobile devices. The success of cellular standards NMT(1G), GSM and CDMA (2G), W-CDMA (3G) and OFDMA (4G) have resulted in creation of market worth billions of dollar. The success of WiFi (Wireless LAN) standard in providing reliable wireless home network has made it an essential part of day to day life. According to reports, in 2013, the global mobile traffic reached 1.5 exabytes per month and total mobile devices exceeded 6.5 billion. This explosive growth has led to development of wireless network architectures where transmissions by neighboring nodes can be cooperative rather than just treating it as interference.

Wireless networks suffer from various impairments like additive white Gaussian noise, received power reduction due to path loss, fading etc. Fading is defined as random fluctuations in received signal amplitude due to constructive and destructive interference of the signal at receiver antenna. Multiple Input Multiple Output (MIMO) is the technique of using multiple antennas at both the transmitter and receiver to combat fading efficiently and thereby improving communication performance. In addition to MIMO providing spatial diversity, various diversity techniques in time and frequency have been proposed to minimize the detrimental affects of fading[1]. Essentially all diversity techniques rely on providing destination with multiple replicas of transmitted signal which are affected by independent fading. Since, it's less probable that all the independent fading channels will simultaneously in deep fading, hence performance can be improved by using diversity techniques. In a power constrained networks, however, its not always possible to use multiple antennas at transmitter and/or receiver.

1.1 Cooperative Communication

Cooperative communication has emerged as an important technology for improving the throughput of wireless networks. It is a technique by which single antenna sources cooperate to create virtual MIMO like system and reap diversity benefits of MIMO systems[2; 3; 4; 5]. The independent copies of same signal are decoded at the destination resulting in efficient resistance to performance degradation caused by fading. A simple cooperative system is shown in Fig. 1.1.

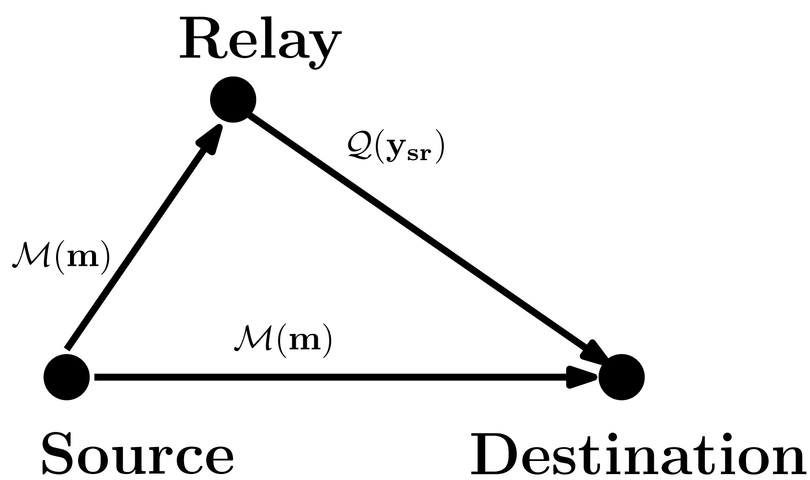

Figure 1.1 A simple cooperative communication system

It consists of a source(S), a relay(R) and a destination(D). In cooperative networks, transmission typically occurs in two phases. In first phase the source sends its message m to the destination. Because of the broadcast nature of the wireless medium, the relay overhears the signal transmitted by the source. The received signal at the destination and the relay after the end of first phase of transmission is given by

$$y_{sd} = \sqrt{E_s} h_{sd} \mathcal{M}(m) + n_{sd} \tag{1.1}$$

$$y_{sr} = \sqrt{E_s} h_{sr} \mathcal{M}(m) + n_{sr} \tag{1.2}$$

where h_{sd}, h_{sr} are the channel gain from source to destination and source to relay respectively. They are modeled as complex Gaussian random variable with mean zero and variances $d_{sd}^{-\alpha}$, $d_{sr}^{-\alpha}$ respectively, α is the path loss exponent, d_{sd} is the distance between source and destination, d_{sr} is the distance between source and relay. $\mathcal{M}(m)$ denotes the modulated signal transmitted by the source. n_{sd}, n_{sr} denote the additive white Gaussian noise at the destination and relay with zero mean and variance $N_0/2$ per dimension. It is assumed that source and relay transmit same energy, $E_s = E_r = E$. In second phase the relay assists the source by re-transmitting source's signal according to a certain protocol to destination. Essentially the relay reprocesses the received signal from the source and transmit it to the destination which can be modeled as

$$y_{rd} = h_{rd}\mathcal{Q}(y_{sr}) + n_{rd} \tag{1.3}$$

where y_{rd} is the received signal at the destination, $\mathcal{Q}(.)$ is the function utilized by the relay to process the received signal. h_{rd} is the channel gain from relay to destination modeled as complex Gaussian random variable with mean zero and variances $d_{rd}^{-\alpha}$. n_{rd} denotes the additive white Gaussian noise at the destination with zero mean and variance $N_0/2$ per dimension. Several protocols have been proposed in the literature for relay's operation in second phase. The notable ones are[6; 7]

- **Amplify and Forward (AF)**: In the AF protocol, the relay node amplifies the received signal and forwards the amplified version to the destination. The amplification gain at the relay node is chosen according to the relay power constraint. The signal transmitted by the relay is given by

$$\mathcal{Q}(y_{sr}) = \beta y_{sr} \tag{1.4}$$

  where β is the amplification provided by the relay node. For fulfilling the power constraint, the amplification provided by the relay node is given by

$$\beta = \sqrt{\frac{E}{|h_{sr}|^2 E + N_0}} \tag{1.5}$$

  The received signal by the destination in the second phase is given by

$$y_{rd} = h_{rd}\sqrt{\frac{E}{|h_{sr}|^2 E + N_0}} y_{sr} + n_{rd} \tag{1.6}$$

$$= \frac{E}{\sqrt{|h_{sr}|^2 E + N_0}} h_{rd} h_{sr} \mathcal{M}(m) + \sqrt{\frac{E}{|h_{sr}|^2 E + N_0}} n_{sr} + n_{rd} \tag{1.7}$$

The destination receives two copies of m through two different paths. It applies maximal ratio combining (MRC) to decode m as given by following equation

$$y = a_1 y_{sd} + a_2 y_{rd} \tag{1.8}$$

where a_1 and a_2 are the scaling factors chosen to maximize the combined signal to noise ratio (SNR). They are given by

$$a_1 = \frac{\sqrt{E} h_{sd}^*}{N_0}, \qquad a_2 = \frac{\frac{E}{\sqrt{|h_{sr}|^2 E + N_0}} h_{rd}^* h_{sr}^*}{N_0'} \tag{1.9}$$

where $N_0' = N_0 \left(1 + \frac{E|h_{rd}|^2}{E|h_{sr}|^2 + N_0}\right)$. The resultant SNR, γ_{MRC}, can be expressed as

$$\gamma_{MRC} = \gamma_1 + \gamma_2 \tag{1.10}$$

where

$$\begin{aligned} \gamma_1 &= \frac{E|h_{sd}|^2}{N_0} \\ \gamma_2 &= \frac{1}{N_0} \frac{E^2 |h_{rd}|^2 |h_{rs}|^2}{E(|h_{sr}|^2 + |h_{rd}|^2) + N_0} \end{aligned} \tag{1.11}$$

The maximum average mutual information between the input and the MRC output, achieved by Gaussian distributed input code-books, is given by

$$I_{AF} = \frac{1}{2} \log\left(1 + \gamma_{MRC}\right) = \frac{1}{2} \log\left(1 + \Gamma|h_{sd}|^2 + f(\Gamma|h_{sr}|^2, \Gamma|h_{rd}|^2)\right) \tag{1.12}$$

where $\Gamma = \frac{E}{N_0}$ and

$$f(x, y) \triangleq \frac{xy}{x + y + 1} \tag{1.13}$$

At high SNR and for Rayleigh fading, the outage probability of AF for spectral efficiency R is given by

$$P_{out,AF} = Pr(I_{AF} < R) \approx \left(\frac{2^{2R} - 1}{\Gamma}\right)^2 \tag{1.14}$$

From above equation, we can clearly see that the AF protocol achieves diversity order of 2.

- **Decode and Forward (DF)**: In the DF protocol, the relay node decodes the received signal, re-encodes it, and transmits the encoded signal to the destination. Let the decoded signal at the relay be represented as $\hat{m}$. The received signal at the destination is given by

$$y_{rd} = h_{rd}\sqrt{E}\mathcal{M}(\hat{m}) + n_{rd} \tag{1.15}$$

The processing at the relay is given by

$$\mathcal{Q}(y_{sr}) = \mathcal{M}(\hat{m}) \tag{1.16}$$

The processing at relay might be full decoding the whole codeword and re-encoding it for transmission or it might be Minimum Mean Square Error (MMSE) estimates of transmitted symbols. Depending on various types of processing at the relay, tradeoff between complexity and performance can be achieved. Assuming complete decoding and repetition re-encoding at the relay, the maximum average mutual information is given by

$$I_{DF} = \frac{1}{2}\min\{\log(1+\Gamma|h_{sr}|^2), \log(1+\Gamma|h_{sd}|^2 + \Gamma|h_{rd}|^2)\} \tag{1.17}$$

This is because the end-to-end mutual information of DF protocol is limited by the mutual information of the weakest link between the source-relay and the combined channel from the source-destination and relay-destination. The outage probability for DF protocol is given by

$$P_{out,DF} = Pr(I_{DF} < R) = Pr\left(\min\{|h_{sr}|^2, |h_{sd}|^2 + |h_{rd}|^2\} < \frac{2^{2R}-1}{\Gamma}\right) \tag{1.18}$$

This can be expressed as

$$P_{out,DF} = Pr\left(|h_{sr}|^2 < \frac{2^{2R}-1}{\Gamma}\right) + Pr\left(|h_{sr}|^2 > \frac{2^{2R}-1}{\Gamma}\right) Pr\left(|h_{sd}|^2 + |h_{rd}|^2 < \frac{2^{2R}-1}{\Gamma}\right) \tag{1.19}$$

At high SNR, the outage probability can be simplified as follows

$$P_{out,DF} \approx \frac{2^{2R}-1}{\Gamma} \tag{1.20}$$

The diversity order of DF is limited to one since the requirement of relay to fully decode the source information puts limitation on overall performance achieved by the system to be limited by the weakest link from source to relay and source to destination.

- **Selection Relaying (SR)**: The performance of DF protocol is limited by the source-relay link. In order to overcome this shortcoming, selection relaying protocol can be used. In this scheme, if the measured source-relay channel strength, $|h_{sr}|^2$, is below certain threshold, then the source continues to transmit to the destination using repetition code or any other channel code in second phase. If the measured source-relay channel strength is above certain threshold, then the relay forwards what it received from source in first phase either using AF or DF. The maximum mutual information for selection relaying protocol can be shown to be

$$I_{SR} = \begin{cases} \frac{1}{2}\log\left(1 + 2|h_{sd}|^2\Gamma\right) & |h_{sr}|^2 < \frac{2^{2R}-1}{\Gamma} \\ \frac{1}{2}\log\left(1 + \left(|h_{sd}|^2 + |h_{rd}|^2\right)\Gamma\right) & |h_{sr}|^2 \geq \frac{2^{2R}-1}{\Gamma} \end{cases} \tag{1.21}$$

  At high SNR and for Rayleigh fading, the outage probability of SR for spectral efficiency R is given by

$$P_{out,SR} = Pr(I_{SR} < R) \approx \left(\frac{2^{2R}-1}{\Gamma}\right)^2 \tag{1.22}$$

  From above equation, we can clearly see that the SR protocol achieves diversity order of 2.

- **Compress and Forward (CAF)**: The CAF is the cooperative protocol which allows the relay node to compress the received signal from the source node and forward it to the destination without decoding the signal. The Wyner-Ziv coding can be used for optimal compression.

1.2 Cooperative Coding

Cooperative Coding[9; 10] is a method that merges ideas of channel coding and cooperative communication. The idea of coded cooperation is to use the same overall rate for coding and transmission, however, the coded symbols are re-arranged between the users such that better diversity is attained. Suppose that there are two users in the system, communicating to a common destination as shown in Fig. 1.2. Each user has K information bits and it transmits N coded bits so that code rate $R = K/N$. Under coded cooperation each coded packet of length N is divided into two parts of length N_1 and N_2 respectively where $N_1 + N_2 = N$.

In first phase, a codeword of higher rate $R_1 = K/N_1$ is broadcasted by source 1. If source 2 can successfully decode information bits of source 1, then it re-encodes them as N_2 bits and transmits them. The destination receives two noisy codewords which it can combine to form a low rate code to decode K information bits of source 1. If source 2 is not able to successfully decode the K information bits of source 1, then it transmits N_2 coded bits of its own data and the same process repeats. After the second phase of communication, a total of N bits are transmitted for K information bits of each user. The two parts of codeword are received at the destination through independent fading channels. The performance of this method shows good gains in slow Rayleigh fading, even when the inter-user channel is much worse than the source-destination channel.

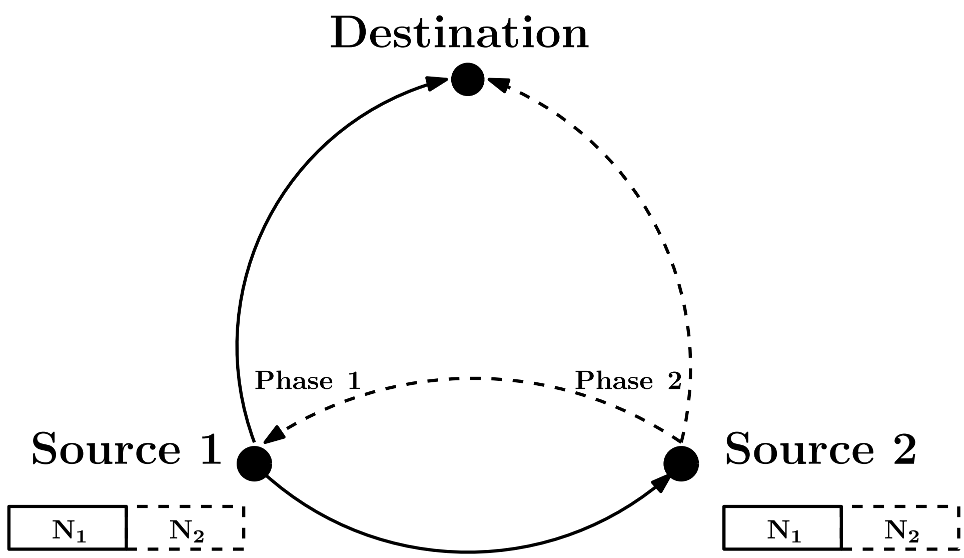

Figure 1.2 Cooperative coding system model

1.3 Network Coding

Network Coding [8] is a technique in which, instead of simply forwarding the packets, the intermediate nodes of a network combine several received packets to generate one or multiple output packets and transmit them to other nodes. This can be used to improve the information flow in a network. Linear network coding has been proved to achieve multi-cast capacity of a network.

Consider a three node wireless network as shown in Fig.1.3. Here node A and B wants to exchange their respective message packets $\mathbf{m}_A$ and $\mathbf{m}_B$ through the relay node R. In case of traditional scheme, first node A broadcasts its packet to R and then node B broadcasts its packet to R. Node R, after receiving both packets, broadcasts $\mathbf{m}_A$ and $\mathbf{m}_B$ to node B and node A respectively. This entire packet exchange takes four time slots. In case of network coding, instead of broadcasting packet $\mathbf{m}_A$ and $\mathbf{m}_B$ in third and fourth time slots, the relay can broadcast $\mathbf{m}_r = \mathbf{m}_A \oplus \mathbf{m}_B$. Since node A already has $\mathbf{m}_A$, it can decode $\mathbf{m}_B$ by xoring $\mathbf{m}_A$ with $\mathbf{m}_r$. Similarly node B can decode $\mathbf{m}_A$. Hence by using network coding, the packet exchange takes only three time slots instead of four, thereby improving network throughput.

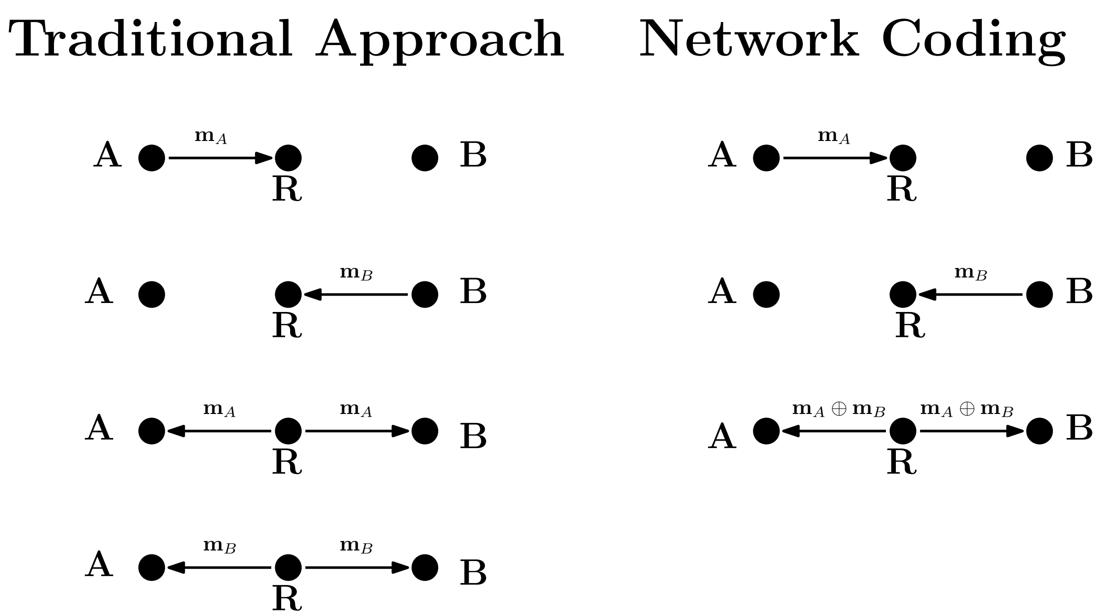

Figure 1.3 Two way network coding system model

Next consider the *butterfly network* of Fig. 1.4. Here sources S_1 and S_2 both want their packet m_1 and m_2 to reach both destinations D_1 and D_2. All the links in the network are of unit capacity. In first time slot, S_1 transmits m_1 which is received successfully by D_1 and R_1. In the second time slot, S_2 transmits m_2 which is successfully received by D_2 and R_1. Under traditional routing scheme, in third and fourth time slots, R_1 forwards m_1 and m_2 to R_2 which are in turn forwarded to D_2 and D_1 in fifth and sixth time slots respectively. When using networking coding, R_1 can forward $m_r = m_1 \oplus m_2$ to R_2 and R_2 can forward m_r to D_1 and D_2 in fourth time slot. Since D_1 already has successfully received m_1, it can decode m_2 from m_r and similarly D_2 can decode m_1. Hence using network coding, the destinations are able to receive both m_1 and m_2 in four time slots instead of six. This is a huge improvement in throughput of the network.

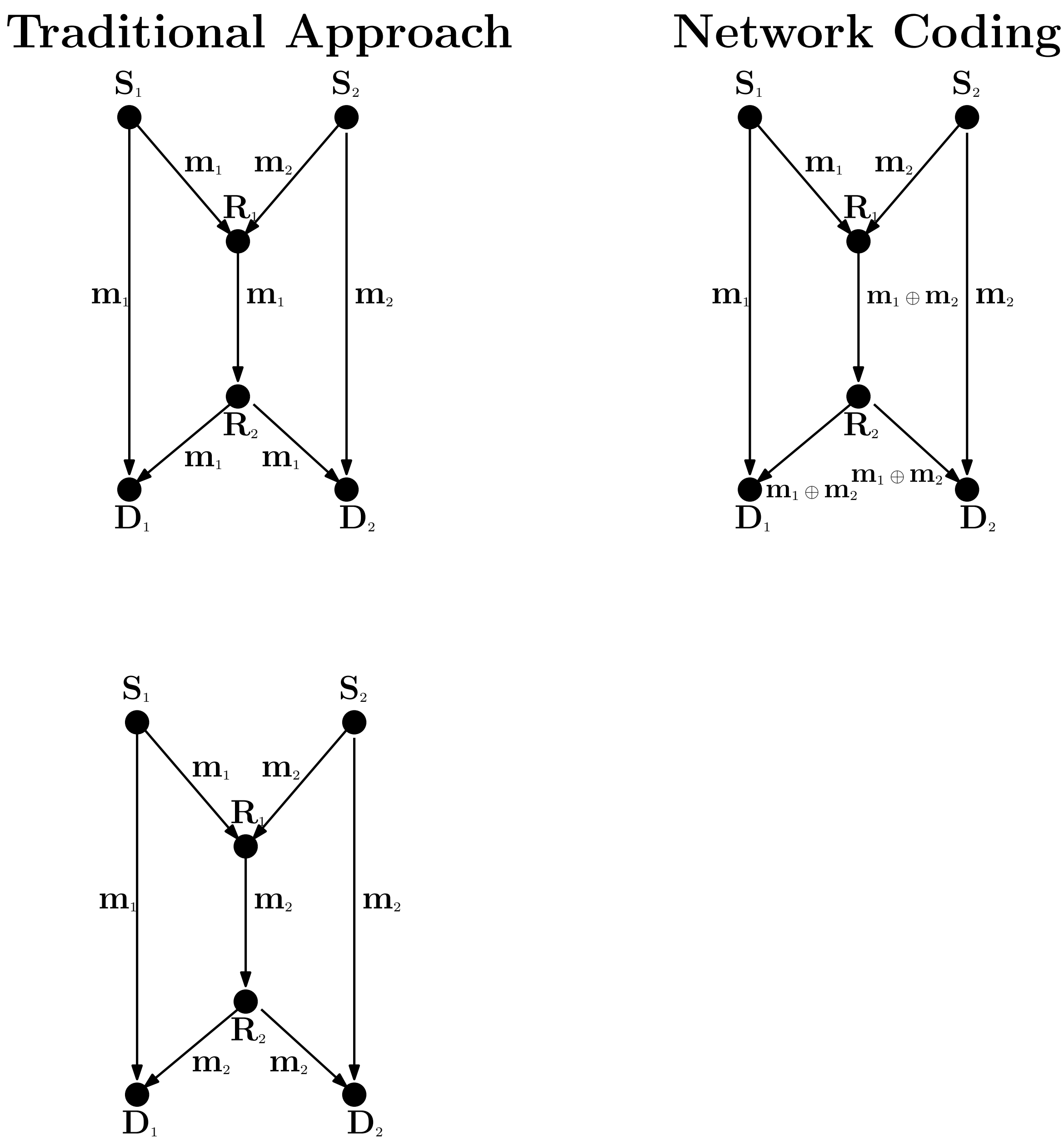

Figure 1.4 Butterfly Network

In addition to improving throughput, network coding has been shown to enhance the robustness of the network, improves security and enables distributed data sharing in the network.

1.4 Multiple Access Relay Network

In multiple-access relay network (MARN), multiple sources communicate with a single or multiple destinations in the presence of relay nodes. These relay nodes can be either dedicated nodes or sources can take turn to act as relay for other sources. Examples of such networks include hybrid wireless LAN/WAN networks and sensor and ad-hoc networks etc. Various multiple access techniques - frequency division multiple access (FDMA), time division multiple access (TDMA), code division multiple access (CDMA) etc. can be used to coordinate the transmissions on common medium. Typically each wireless node is assigned unique frequency or time slot or code for its transmission. In multiple source networks, in addition to providing a diversity and/or multiplexing gain, relay nodes can provide other tasks such as mitigating the interference effect among sources, maximizing the signal to noise ratio, and minimizing the mean square error. The intermediate relay nodes can also apply techniques such as network coding, cooperative coding to improve throughput of the network.

When multiple sources transmit simultaneously, zero forcing (ZF) relaying can be used in which the interference among sources can be completely removed by adjusting the weights at relay nodes [61; 62; 63; 64]. Another technique to improve SNR at the destination is minimum mean square error (MMSE) relaying where the weights of relay nodes are adjusted to minimize the mean square error between the source signal and the relayed signal to the destination [65; 66; 67]. Coherent relaying, QR decomposition relaying, and distributed beamforming relaying proposed in [68; 69; 70] are some other examples of relaying schemes in multiple access relay networks.

1.5 Relay Selection

Cooperative communication schemes provide considerable gain in fading wireless environments. Using protocols like AF, DF or by using distributed space-time algorithms, harmful effects of fading can be efficiently minimized. However as the network size grows, precise synchronization and distributed operation of the wireless nodes becomes increasingly difficult. Also the design of distributed space time codes becomes complex. Relay selection is a scheme that selects the best relay between source and destination based on *instantaneous* channel measurements. Consider a cooperative network with one source S, N relays R_1, R_2,..., R_N and one destination Das shown in Fig.1.5. In the first time slot, source S broadcasts its message m to the N relays. The message m is received at the destination and by the relays. In second time slot, a single *best* relay is chosen to retransmit source signal to the destination. The best relay is chosen such that the combined SNR at the destination is maximized. When the relays employs amplify and forward scheme the best relay is selected as [71]

$$\gamma_{AF} = \gamma_{SD} + \arg \max_{i \in \{1,2,\ldots,N\}} \left[\frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i} + \gamma_{R_iD} + 1} \right] \tag{1.23}$$

where γ_{AF} denotes the total SNR at the destination, γ_{SD} denotes the SNR of the direct source-destination link, γ_{SR_i} denotes the source-i^{th} relay SNR and γ_{R_iD} denotes i^{th} relay and destination SNR.

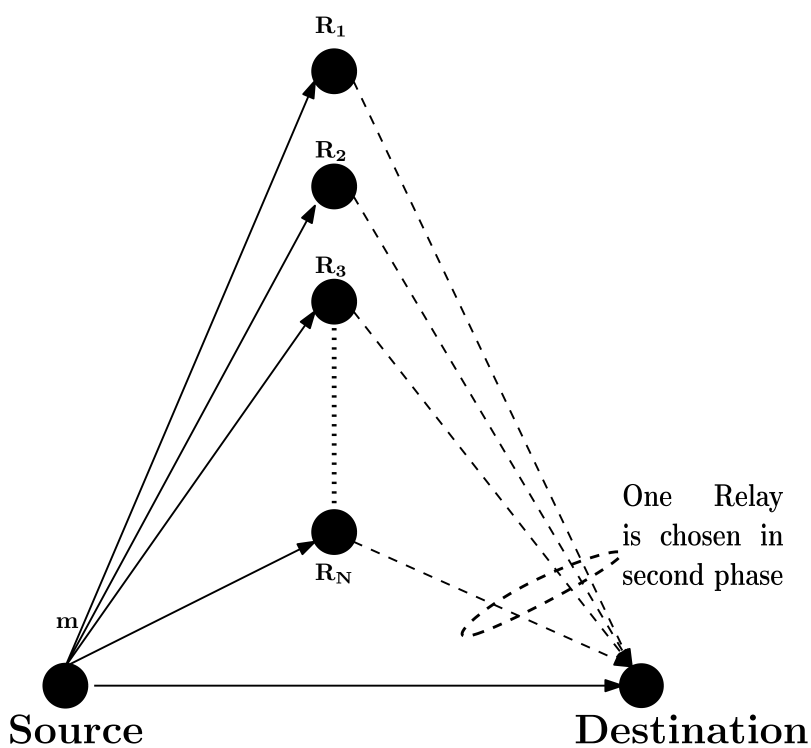

Figure 1.5 Relay selection system model

When the relays employ decode and forward protocol the best relay is selected according to following rule

$$\gamma_{DF} = \gamma_{SD} + \arg \max_{i \in \{1,2,\ldots,N\}} \xi_i \tag{1.24}$$

where ξ_i is the effective SNR of the i^{th} cascaded link $S \to R_i \to D$. The PDF of ξ_i is given by

$$f_{\xi_i}(x) = B_i \delta(x) + (1 - B_i) \frac{1}{\overline{\gamma}_{R_i D}} \exp\left(-\frac{x}{\overline{\gamma}_{R_i D}}\right) \tag{1.25}$$

with

$$\begin{aligned} B_i =& a \int_0^\infty \operatorname{erfc}\left(\sqrt{b\gamma}\right) f_{\gamma_{SR_i}}(\gamma) d\gamma \\ =& a \left[\frac{b\overline{\gamma}_{SR_i}}{b\overline{\gamma}_{SR_i} + 1}\right] \end{aligned} \tag{1.26}$$

where a and b are the constants given by type of modulation (eg. for BPSK b=1 and a = 1/2), $\overline{\gamma}_{SR_i} = d_{SR_i}^{-\alpha}\gamma$, $\overline{\gamma}_{R_i D} = d_{R_i D}^{-\alpha}\gamma$, $\gamma = E_s/N_0$ and erfc(x) is the complementary error function given by

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) dt \tag{1.27}$$

The asymptotic bit error probability when using AF protocol is given by

$$P_e \approx \frac{1}{M} \frac{\Gamma(M + 3/2)}{\sqrt{\pi} M (M+1) B^{M+1}} \left(\frac{1}{\overline{\gamma}_{SR_i}} + \frac{1}{\overline{\gamma}_{R_i D}}\right)^N \frac{1}{\overline{\gamma}_{SD}} \tag{1.28}$$

where $\Gamma(x)$ is the gamma function. Clearly the diversity order of the scheme is $N + 1$ which is the maximal achievable diversity order with N relays.

1.6 Thesis Organization

The remainder part of thesis is organized as follows. In Chapter 2, we present network coding scheme to provide differentiated QoS to different users. The system model is described in Section 2.2, the scheme is proposed in Section 2.3, bit error and outage probability are derived in Section 2.4, parameter optimization is described in Section 2.5 and numerical results are presented in Section 2.6.

In Chapter 3, we present iterative packet recycling scheme to mitigate falsified injection. The system model and the attack model assumptions are described in Section 3.2. We then briefly describe the Chase-Pyndiah decoding algorithm used for decoding at the destination in

Section 3.3. Under the assumed first attack model, the packet recycling scheme to detect and mitigate false injection is described in Section 3.4. The simulation results for packet recycling are presented in Section 3.5.

In Chapter 4, we present relay selection scheme to improve network lifetime. The system model is described in Section 4.2 and the scheme is proposed in Section 4.3. We then derive outage probability, relay usage and relay switching rates for proposed scheme and conventional scheme in Section 4.4. Finally we present numerical results in Section 4.5.

Finally, the conclusion and the future work are discussed in Chapter 5.

CHAPTER 2. SOFT PRIORITIZED NETWORK CODING

2.1 Introduction

Cooperative communication has emerged as an important technology for satisfying growing demand of wireless services. In this scheme, one or more wireless nodes act as relay to facilitate communication between other nodes. Various cooperative protocols and their variations have been proposed and analyzed in the literature, notable ones being Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CF) [24; 25; 26; 7]. Various relay nodes of the network cooperate to form a virtual antenna array. This exploits spatial diversity amongst cooperating nodes which improves the overall network performance.

Network Coding (NC) is another efficient solution proposed to improve spectral efficiency for wired communication [8]. The basic idea is that the intermediate relay nodes encode the incoming packets such that they can be reliably decoded at their destination. Due to broadcast nature of wireless medium, the intermediate relay nodes can overhear data from other source nodes. Hence network coding can be naturally combined with cooperative communication. Various network coded cooperation techniques have been proposed in the literature where the destination jointly decodes the information received from intended sources and relays[35; 36; 37]. Many network code designs assume that all the sources involved have uniform QoS requirements whereas in many practical communication scenarios, some nodes may be more important (prioritized) than others and hence require a higher QoS than others. In such cases providing a uniform QoS to all sources may be either a wasteful or an infeasible approach.

Several network coding techniques taking the priority of source nodes into account have been developed in [11; 12; 13; 14; 19]. In [11], Lin et.al proposed linear coding schemes, Stacked and Progressive linear codes, for P2P and sensor networks. In [12], Silva et.al proposed priority encoding using rank-metric codes. In [13], Limmannee et.al designed the global encoding kernels (GEKs) to provide unequal error protection. In [14], Thomos et.al proposed use of random linear network codes for efficient video delivery which are constructed in such a way that important packets are included more often in network encoding. However all these papers assume erasure and binary symmetric channels and do not consider channel variations in designing codes. Therefore, these techniques can provide only a fixed (hard) level prioritized service: if a source is included in the network encoder it is assisted by the relay, otherwise, no assistance is provided. In recent works [19; 29], authors have proposed providing variable QoS to source nodes by using UEP codes as network codes on multiple relays. Their scheme relies on assigning different diversity orders to different sources depending on their respective individual QoS requirements. This, however, leads to a scheme in which non-prioritized user suffers reduction in respective diversity order and hence reduction in overall throughput of the network.

We first consider a two-hop multiple access relay network where two source nodes communicate with a common destination with the assistance of a relay. A naive variable QoS coding scheme would allow the relay to transmit the networked coded packet for two users for some time and the data for prioritized user for the remaining time. A quick analysis of this naive scheme proves that the non-prioritized user suffers from diversity loss if the coded packets are generated at random time instances. We identify those *opportunities* when relay should transmit the network coded packet for two nodes or the packet of prioritized user such that the highest diversity order can be provided for both prioritized and non-prioritized user. The basic idea is to exclude the non-prioritized user in the network encoding when its channel gain towards the destination is above a threshold (good enough). The threshold can be adapted according to the QoS requirement of the users. As shown by simulation and theoretical analysis, the proposed scheme can provide a variable QoS and still maintain the maximal diversity order for both prioritized and non-prioritized nodes. We derive outage probability for the prioritized and non-prioritized users in Rayleigh flat fading and additive white Gaussian noise (AWGN). We also derive the optimum threshold and relay location that provide a given QoS requirement. We then generalize this scheme for multi-source multi-relay scenario and propose the encoding rule at various relays.

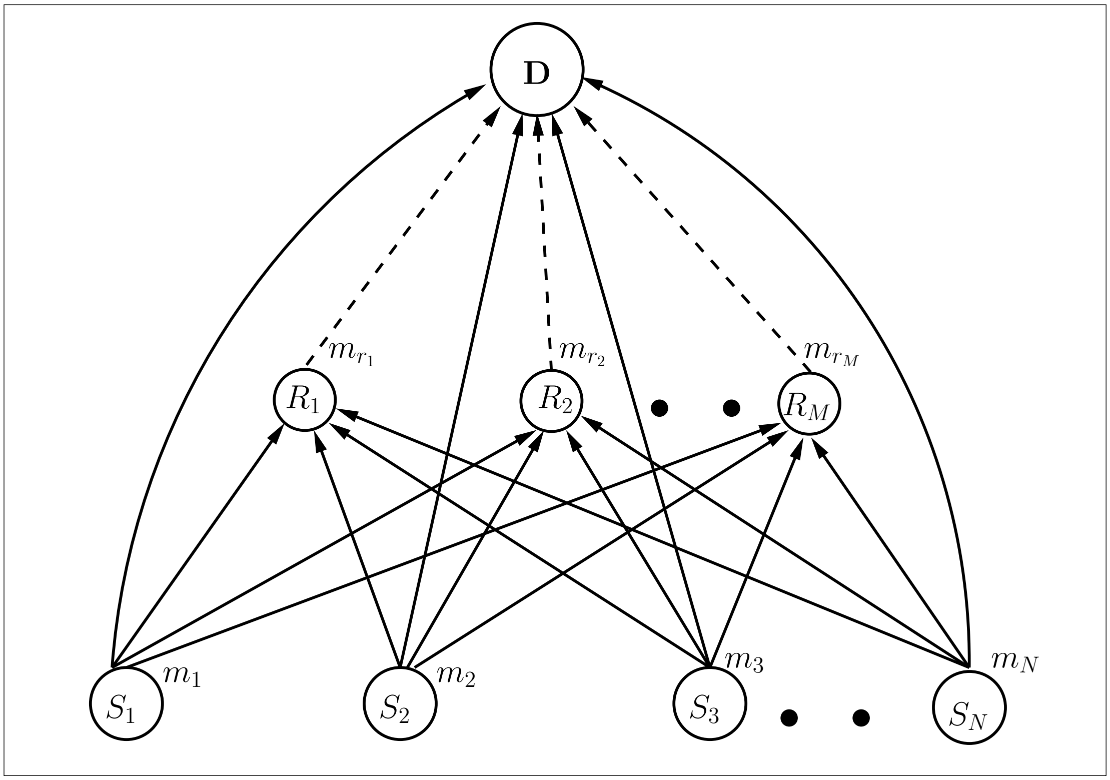

Figure 2.1 Multiple Access Relay Network for N sources, M relays and single destination

2.2 System Model

Consider a multiple access relay network, shown in Fig. 2.1, where the source nodes S_i, $i = 1, 2, .., N$ transmit independent message packets $\mathbf{m}_i$, $i = 1, 2,..,N$ to a common destination D through orthogonal channels (in time or frequency). The relay nodes R_j, $j = 1, 2, ..., M$ after overhearing source packets generates network coded packets $\mathbf{m}_{r_j}$ and transmits them to D using q-ary modulation through orthogonal channels (in time or frequency). Hence N message packets are sent to D in $N + M$ channel uses. The traditional NC rule is $\mathbf{m}_{r_j} = \bigoplus_{i=1}^{N} \beta_i \ \mathbf{m}_i$, where $\oplus$ denotes modulo operation in GF(q) and β_i are network coding coefficients. To avoid error propagation, we assume that only the correctly decoded packets (indicated by CRC) are included in generating the coded packet at the relay. If only one packet is decoded correctly, then the relay forwards the correctly decoded packet only. If all the source packets are received incorrectly at a relay, then it remains silent. Let

$$\mathbf{y}_i = h_i \sqrt{E_i} \mathcal{X}(\mathbf{m}_i) + \mathbf{n}_{id}, \quad i \in \{1, 2..., N\} \tag{2.1}$$

denote the received signal at D from source node S_i. $\mathcal{X}(\mathbf{m}_i)$ is the Gaussian distributed coded packet for $\mathbf{m}_i$. h_i is the channel gain between the i^{th} source node and the destination D, modeled as $\mathcal{CN}\left(0, d_{id}^{-\alpha}\right)$, where $\mathcal{CN}(0,1)$ denotes the complex Gaussian distribution with mean 0 and variance 1, α is the path loss exponent, and d_{id} is the distance between the i^{th} source node and the destination. E_i is the transmit energy from source node S_i. $\mathbf{n}_{id}$ is the AWGN noise vector, modeled as $\mathcal{CN}(0, N_0)$. Let

$$\mathbf{z}_j = h_{r_j} \sqrt{E_{r_j}} \mathcal{X}(\mathbf{m}_{r_j}) + \mathbf{n}_{r_j d}, \quad j \in \{1, 2..., M\} \tag{2.2}$$

denote the received signal at D from relay node R_j. h_{r_j} is the channel gain between the j^{th} relay node and the destination, modeled as $\mathcal{CN}\left(0, d_{r_j d}^{-\alpha}\right)$, where $d_{r_j d}$ is the distance between the j^{th} relay node and the destination. E_{r_j} is the transmit energy from j^{th} relay node S_i and $\mathbf{n}_{r_j d}$ is the AWGN noise vector, modeled as $\mathcal{CN}(0, N_0)$. Let d_{ir_j} is the distance between the i^{th} source node and j^{th} relay node. We assume transmit energies to be equal from all source and relay nodes i.e. $E_i = E_{r_j} = E_s \; \forall \; i \in \{1, 2..., N\}$, $j \in \{1, 2..., M\}$.

2.3 Channel Adaptive Prioritized Network Coding

2.3.1 Two users, Single relay

In this section we will describe the proposed scheme. First we will derive and propose the scheme using Log-likelihood ratios (LLR) for two sources, single relay multiple access relay network. To get insight in our proposed scheme, first we will assume BPSK modulated codewords.

Consider a MARN consisting of two sources S_1 and S_2 communicating with a common destination D by the help of single relay R_1 as shown in Fig. 2.2. Without loss of generality suppose that S_1 is of higher priority than S_2. To get insight into our scheme, we assume that BPSK modulation is used. Consider first the case where both $\mathbf{m}_1$ and $\mathbf{m}_2$ are decoded correctly by the relay and the coded packet $\mathbf{m}_{r_1} = \mathbf{m}_1 \oplus \mathbf{m}_2$ is sent to D. Let m_i^j denote the j^{th} bit of

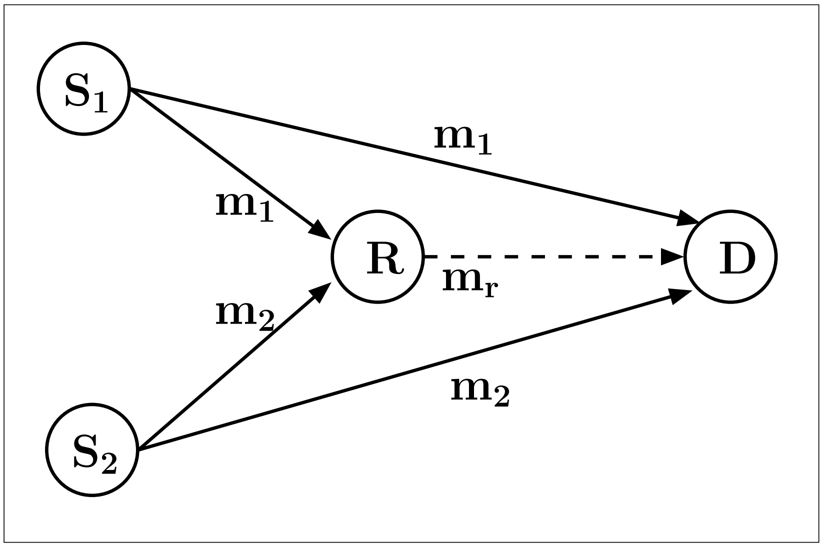

Figure 2.2 System Model for MARN with two sources, single relay, single destination.

$\mathbf{m}_i$ and y_i^j denote the j^{th} received bit of $\mathbf{y}_i$, $i = 1, 2, r_1$. Then, the log likelihood ratio (LLR) of m_2^j at D given $\mathbf{h} = (h_1, h_2, h_{r_1})$ and $\mathbf{y}^j = (y_1^j, y_2^j, y_{r_1}^j)$ is given by [20]

$$L(m_2^j|\mathbf{h}, \mathbf{y}^j) = L(m_2^j|h_2, y_2^j) + L(m_{r_1}^j \oplus m_1^j|h_1, y_1^j, h_{r_1}, y_{r_1}^j) \tag{2.3}$$

where $L(m_2^j|h_2, y_2^j)$ is the LLR of m_2^j provided by the direct $\mathbf{S}_2$-D link and $L(m_{r_1}^j \oplus m_1^j|h_1, y_1^j, h_{r_1}, y_{r_1}^j)$ is the additional information provided by the R-D link. The reliability of decision on m_2^j is determined by the magnitude of $L(m_2^j|\mathbf{h}, \mathbf{y}^j)$ which is bounded by

$$|L(m_2^j|\mathbf{h}, \mathbf{y}^j)| \leq |L(m_2^j|h_2, y_2^j)| + |L(m_{r_1}^j \oplus m_1^j|h_1, y_1^j, h_{r_1}, y_{r_1}^j)| \tag{2.4}$$

where the second term can be approximated as [20]

$$|L(m_r^j \oplus m_1^j|h_1, y_1^j, h_{r_1}, y_{r_1}^j)| \approx \min\{|L(m_{r_1}^j|h_{r_1}, y_{r_1}^j)|, |L(m_1^j|h_1, y_1^j)|\} \tag{2.5}$$

Here

$$L(m_i^j|h_i, y_i^j) = \frac{4\sqrt{E_i}}{N_0}\Re e\{h_i^* y_i^j\} \tag{2.6}$$

where $\Re e\{\cdot\}$ denotes the real part of a complex number and h_i^* denotes the complex conjugate of h_i. Clearly the sign of $L(m_2^j|\mathbf{h}, \mathbf{y}^j)$ is determined by $L(m_2^j|h_2, y_2^j)$ if the following condition holds

$$|L(m_2^j|h_2, y_2^j)| > \min\{|L(m_{r_1}^j|h_{r_1}, y_{r_1}^j)|, |L(m_1^j|h_1, y_1^j)|\} \tag{2.7}$$

This means that the MAP detection of m_2^j does not depend on the additional information provided by the relay if Eq.(2.7) is satisfied. Hence the relay can choose not to include m_2^j in $m_{r_1}^j$ without affecting its reliability at the destination when Eq.(2.7) is true. The magnitude of LLR when averaged over the Gaussian noise is given by

$$E[|L(m_i^j|h_i, y_i^j)|] = 4|h_i|^2\gamma_i + 4\sqrt{\frac{\gamma_i|h_i|^2}{\pi}}e^{-\gamma_i|h_i|^2} - 8\gamma_i|h_i|^2 Q\left(\sqrt{2\gamma_i|h_i|^2}\right) \tag{2.8}$$

where $\gamma_i = E_i/N_0$ and $Q(x)$ denotes the complementary unit Gaussian distribution function $\frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$. The derivation to Eq.(2.8) is given in Appendix A.

Since $E[|L(m_i^j|h_i, y_i^j)|] \approx 4|h_i|^2\gamma_i$ and this approximation is fairly accurate even for low γ_i, we propose the following network coding rule, assuming $E_1 = E_2 = E_{r_1} = E_s$:

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{m}_1 \oplus \mathbf{m}_2 & \text{if } |h_2|^2 < \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_2|^2 \geq \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \end{cases} \tag{2.9}$$

for some constant μ which can be adjusted to provide a variable QoS to the source nodes, $0 < \mu \leq 1$. We will discuss how to optimize μ later. The above rule henceforth will be referred to as *soft prioritized network coding* (SPNC) rule. The encoding rule in Eq.(2.9) can be implemented in practice by having the destination determine the encoding based on the estimated channel gain $|h_1|^2$, $|h_2|^2$, $|h_{r_1}|^2$ and informing the relay ($\mathbf{m}_{r_1} = \mathbf{m}_1$ or $\mathbf{m}_1 \oplus \mathbf{m}_2$) using 1-bit feedback piggybacked on clear to send (CTS) transmitted to the relay.

2.3.2 Three users, Single relay

The proposed SPNC scheme can be easily extended to three, single relay case. Suppose the three users, S_1, S_2 and S_3 requiring different class of service such that S_1 is prioritized and S_2 and S_3 are non-prioritized. Suppose they are communicating with a common destination D by the help of single relay R_1. We will describe the SPNC scheme for this case assuming that all three users have been successfully at the relay.

Since S_1 is of highest priority, hence $\mathbf{m}_1$ will always be included in network coded packet $\mathbf{m}_r$. Now it needs to be decided whether to include $\mathbf{m}_2$ and $\mathbf{m}_3$ or not in the coded packet. This can be performed similarly as described in previous section. For deciding whether to include

$\mathbf{m}_2$ in network coded packet or not, we can have following rule

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{m}_1 \oplus \mathbf{m}_2 & \text{if } |h_2|^2 < \mu_2 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_2|^2 \geq \mu_2 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \end{cases} \tag{2.10}$$

Similarly to decide whether to include $\mathbf{m}_3$ in network coded packet or not, we can have following rule

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{m}_1 \oplus \mathbf{m}_3 & \text{if } |h_3|^2 < \mu_3 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_3|^2 \geq \mu_3 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \end{cases} \tag{2.11}$$

where μ_2 and μ_3 can be adjusted to provide required QoS to S_2 and S_3. For example if S_2 is of higher priority than S_3, we choose $\mu_2 > \mu_3$. Combining Eq.(2.10) and (2.11), we have following SPNC rule for 3 users

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{m}_1 \oplus \mathbf{m}_2 \oplus \mathbf{m}_3 & \text{if } |h_2|^2 < \mu_2 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \text{ and } |h_3|^2 < \mu_3 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 \oplus \mathbf{m}_2 & \text{if } |h_2|^2 < \mu_2 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \text{ and } |h_3|^2 > \mu_3 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 \oplus \mathbf{m}_3 & \text{if } |h_2|^2 > \mu_2 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \text{ and } |h_3|^2 < \mu_3 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_2|^2 > \mu_2 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \text{ and } |h_3|^2 > \mu_3 \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \end{cases} \tag{2.12}$$

2.3.3 Two user, Multirelay

The proposed SPNC scheme can be extended to multirelay scenario by using Galois field Network Coding(GFNC) to achieve full diversity for both prioritized and non-prioritized users. In order to generalize the SPNC scheme for multi-relay case, lets discuss the proposed SPNC scheme for two sources and one relay case, Eq.(2.9), to obtain insight for its generalization. From Eq.(2.9), we can see that the destination signals relay to include non-prioritized user's packet in the network coded packet only when the non-prioritized user's channel strength is lower than certain adaptive threshold. This adaptive threshold is function of channel gains of prioritized source and relay channel gain. Using this intuition, we can generalize SPNC rule for multirelay as follows

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{m}_1 \oplus \beta_1 \mathbf{m}_2 & \text{if } |h_2|^2 < \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_2|^2 \geq \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \end{cases} \tag{2.13}$$

where r_1 is the relay closest to the destination. For other relays r_j, $j = 2, 3, ..., M$

$$\mathbf{m}_{r_j} = \begin{cases} \mathbf{m}_1 \oplus \beta_j \mathbf{m}_2 & \text{if } |h_2|^2 < \mu \cdot |h_{r_j}|^2 \\ \mathbf{m}_1 & \text{if } |h_2|^2 \geq \mu \cdot |h_{r_j}|^2 \end{cases} \tag{2.14}$$

where $\beta_j \in GF(2^N)$ for $j = 1, 2, ..., M$ [23; 28; 29]. The reason that we don't include $|h_1|^2$ for relays other than R_1 is because $|h_2|^2$ should be compared with independent thresholds in order for non prioritized user to achieve maximal diversity order. If the thresholds are dependent, then the achievable diversity order for non-prioritized group is reduced. This is proved in Appendix B.

2.3.4 Multiuser, Single relay with Grouping

When the number of sources are large, the number of comparisons of the channel gains increases exponentially for SPNC. In order to reduce the number of comparisons, the sources can be divided in groups based on their class of service demand. For instance there are three class of service - voice, video and data, which require different reliability rate. Hence all sources can be divided in two or three groups according to their class of service requirement and using different scale factors, various level of prioritization can be provided to the users. We propose two approaches to perform grouping. One approach is cluster based approach in which we assume that the users of same class of service are near to each other and can be assumed as a cluster. In the second approach we relax this assumption and the users requiring same class of service can be located anywhere.

2.3.4.1 Cluster based Grouping

Suppose that N users are divided into two groups, say group 1 consisting of $\frac{N}{2}$ sources, S_i, $i = 1, 2, ..., \frac{N}{2}$ and group 2 consisting of other $\frac{N}{2}$ users, S_i, $i = \frac{N}{2} + 1, \frac{N}{2} + 2, ..., N$. The users of each group require same class of service and group 1 is of higher priority than the group 2. Suppose they are communicating with a common destination D by the help of single relay R_1. We assume that group 1 users are clustered together and are at approximately same distance from the destination, say d_1. Similarly group 2 users are clustered together and are at approximately same distance from the destination say d_2 as shown in Fig.2.3.

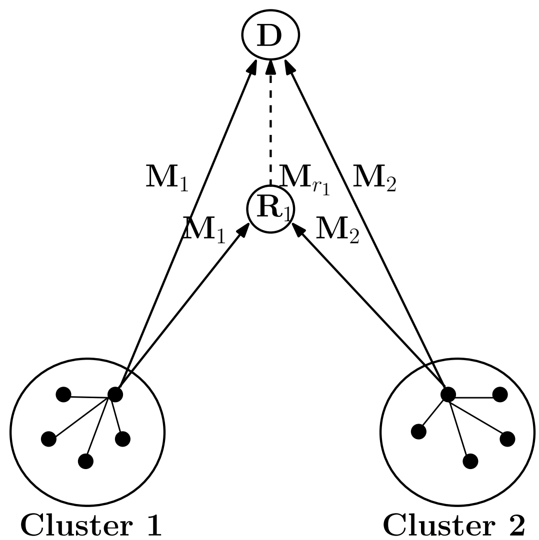

Figure 2.3 Multiuser, Single relay with grouping and clustering

At start of each phase of transmission, the destination chooses group head. The group head can be chosen by destination based on round robin fashion in which each source is chosen once as a group head or randomly from the group. This way no single user is penalized and there is approximately equal energy usage of all the nodes in the group. Now in 1st phase of transmission all the users of the group transmit a message packet, of rate R, to the group head. We assume that all the transmissions in a cluster are error free. Each group head encodes the $\frac{N}{2}$ message packets of rate R in a larger message packet $\mathbf{M}_i$, $i = 1,2$ of rate $\frac{NR}{2}$ and transmits it to the destination. Denoting $G_1 \triangleq |H_1|^2$ as the channel strength of group head for first cluster to destination, $G_2 \triangleq |H_2|^2$ is the channel strength of group head for second cluster to destination and channel gain from relay-destination to be $G_{r_1} \triangleq |h_{r_1}|^2$.

The SPNC coding can be generalized as

$$\mathbf{M}_{r_1} = \begin{cases} \mathbf{M}_1 \oplus \mathbf{M}_2 & \text{if } G_2 < \mu \cdot \min\{G_{r_1}, G_2\} \\ \mathbf{M}_1 & \text{if } G_2 \geq \mu \cdot \min\{G_{r_1}, G_2\} \end{cases} \tag{2.15}$$

2.3.4.2 Normal Grouping

Suppose that N users are divided into two groups, say Group 1 consisting of N_1 sources, S_i, $i = 1, 2, ..., N_1$ and Group 2 consisting of $N_2 = N - N_1$ users, S_i, $i = N_1 + 1, N_1 + 2, ..., N$. The users of each group require same class of service and Group 1 is of higher priority than the Group 2. Suppose they are communicating with a common destination D by the help of single relay R_1. For each group, the weakest source-destination link channel gain is used for comparison. This is done because the weakest source-destination link will have largest outage probability amongst the group. Hence, by considering the weakest source-destination link channel gain for comparison, the relay can provide assistance(via network coding) to the non-prioritized group when the channel gain is below certain adaptive threshold. The parameter, μ, can be suitably chosen such that the weakest user of the group can have certain maximum allowable outage probability. Denoting $|H_1|^2 = \min\{|h_1|^2, |h_2|^2, ..., |h_{N_1}|^2\}$ and $|H_2|^2 = \min\{|h_{N_1+1}|^2, |h_{N_1+2}|^2, ..., |h_N|^2\}$ we have following SPNC rule

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{M}_1 \oplus \mathbf{M}_2 & \text{if } |H_2|^2 < \mu \cdot \min\{|h_{r_1}|^2, |H_1|^2\} \\ \mathbf{M}_1 & \text{if } |H_2|^2 \geq \mu \cdot \min\{|h_{r_1}|^2, |H_1|^2\} \end{cases} \tag{2.16}$$

where $\mathbf{M}_1 \triangleq \bigoplus_{i=1}^{N_1} \mathbf{m}_i$ and $\mathbf{M}_2 \triangleq \bigoplus_{i=N_1+1}^{N} \mathbf{m}_i$.

To implement SPNC, after the first phase of communication, the destination can broadcast a single feedback packet with bits indicating to the relays whether to include non prioritized user or not in the network coded packet. This feedback packet can be piggybacked on CTS sent by destination. In above proposed SPNC scheme, we have assumed that the relays are able to decode all the sources. When some of the sources are decoded incorrectly, then those sources are excluded from network coding at the relay. In case a relay is not able to decode any source successfully, it remains silent.

2.4 Mathematical Analysis

2.4.1 Bit Error Rate for SPNC for two sources, one relay, one destination

In this section, we will derive bit error probabilities for S_1 and S_2 when SPNC scheme is used at the relay. Let $x_1 = |h_1|^2$, $x_2 = |h_2|^2/\mu$, $x_r = |h_{r_1}|^2$. We define following events

- $\mathbf{Z}_1 = \{(x_1, x_2, x_r)|x_1 > x_2 > x_r\}$,
- $\mathbf{Z}_2 = \{(x_1, x_2, x_r)|x_2 > x_1 > x_r\}$,
- $\mathbf{Z}_3 = \{(x_1, x_2, x_r)|x_r > x_2 > x_1\}$,
- $\mathbf{Z}_4 = \{(x_1, x_2, x_r)|x_2 > x_r > x_1\}$,
- $\mathbf{Z}_5 = \{(x_1, x_2, x_r)|x_1 > x_r > x_2\}$,
- $\mathbf{Z}_6 = \{(x_1, x_2, x_r)|x_r > x_1 > x_2\}$

which denote the mutually exclusive events that cover the probability space spanned by random variables x_1, x_2 and x_r.

In the events of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_4$, $x_2 \geq \min\{x_1, x_r\}$ occurs, hence the relay transmits $p = m_1$. In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, $x_2 < \min\{x_1, x_r\}$ occurs, hence the relay transmits $p = m_1 \oplus m_2$. The probability distribution of random variables x_1, x_2 and x_r are given by $\lambda_1 e^{-\lambda_1 x_1}$, $\mu\lambda_2 e^{-\mu\lambda_2 x_2}$ and $\lambda_r e^{-\lambda_r x_r}$, where $\lambda_1 = d_1^{\alpha}$, $\lambda_2 = d_2^{\alpha}$ and $\lambda_r = d_{r_1}^{\alpha}$. These random variables are independent and hence the joint pdf is a product of marginals. The average bit error probability for source S_i, i = 1, 2 is given by

$$P_{e,i} = \sum_{j=1}^{6} P_{e,i,\mathbf{Z}_j} \tag{2.17}$$

where $P_{e,i,\mathbf{Z}_j}$ is the bit error probability for S_i under the event $\mathbf{Z}_j$. In the events of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, the relay transmits $p = m_1$ and the destination then uses MRC to detect m_1. Hence the conditional bit error probability for m_1 given $(\underline{\mathbf{x}}) = \{x_1, x_2, x_r\}$ is given by

$$P_{e,1,\mathbf{Z}_j}(\underline{\mathbf{x}}) = Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) \tag{2.18}$$

where j = 1, 2, 3, 4. Averaging Eq.(2.18) over x_1, x_2 and x_r yields

$$P_{e,1,\mathbf{Z}_j} = \iiint_{\mathbf{Z}_j} P_{e,1,\mathbf{Z}_j}(\underline{\mathbf{x}}) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{2.19}$$

where $f(\underline{\mathbf{x}}) = \mu\lambda_1\lambda_2\lambda_r e^{-(\lambda_1 x_1 + \mu\lambda_2 \cdot x_2 + \lambda_r x_r)}$. Using limit of integration for $\mathbf{Z}_1$ and using Craig's formula [16]

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2sin^2\phi}} d\phi \tag{2.20}$$

we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_1} &= \int_0^{\pi/2}\frac{d\phi}{\pi}\int_0^{\infty}\lambda_1 e^{-x_1\lambda_1(\gamma_1\csc^2\phi+1)}dx_1\int_0^{x_1}\mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2\int_0^{x_2}\lambda_r e^{-x_r\lambda_r(\gamma_r\csc^2\phi+1)}dx_r \\ &= \frac{1}{\pi}\int_0^{\pi/2}\Bigg[\frac{\mu\lambda_2}{(1+\gamma_1\csc^2\phi)\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2\right)}\times \\ &\qquad \frac{\lambda_r}{\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2+(1+\gamma_r\csc^2\phi)\lambda_r\right)}\Bigg]d\phi \end{aligned} \tag{2.21}$$

where $\gamma_1 = d_1^{-\alpha}E_1/N_0$ and $\gamma_r = d_r^{-\alpha}E_r/N_0$ are the receive SNRs at the destination from S_1 and R respectively. Using limit of integration for $\mathbf{Z}_2$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_2} &= \int_0^{\pi/2}\frac{d\phi}{\pi}\int_0^{\infty}\mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2\int_0^{x_2}\lambda_1 e^{-x_1\lambda_1(\gamma_1\csc^2\phi+1)}dx_1\int_0^{x_1}\lambda_r e^{-x_r\lambda_r(\gamma_r\csc^2\phi+1)}dx_r \\ &= \frac{1}{\pi}\int_0^{\pi/2}\frac{\lambda_1\lambda_r}{\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2\right)\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2+(1+\gamma_r\csc^2\phi)\lambda_r\right)}d\phi \end{aligned} \tag{2.22}$$

Using limit of integration for $\mathbf{Z}_3$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_3} &= \int_0^{\pi/2}\frac{d\phi}{\pi}\int_0^{\infty}\lambda_r e^{-x_r\lambda_r(\gamma_r\csc^2\phi+1)}dx_r\int_0^{x_r}\mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2\int_0^{x_2}\lambda_1 e^{-x_1\lambda_1(\gamma_1\csc^2\phi+1)}dx_1 \\ &= \frac{1}{\pi}\int_0^{\pi/2}\Bigg[\frac{\mu\lambda_1}{(1+\gamma_r\csc^2\phi)\left(\mu\lambda_2+(1+\gamma_r\csc^2\phi)\lambda_r\right)}\times \\ &\qquad \frac{\lambda_2}{\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2+(1+\gamma_r\csc^2\phi)\lambda_r\right)}\Bigg]d\phi \end{aligned} \tag{2.23}$$

Using limit of integration for $\mathbf{Z}_4$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_4} &= \int_0^{\pi/2}\frac{d\phi}{\pi}\int_0^{\infty}\mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2\int_0^{x_2}\lambda_r e^{-x_r\lambda_r(\gamma_r\csc^2\phi+1)}dx_r\int_0^{x_r}\lambda_1 e^{-x_1\lambda_1(\gamma_1\csc^2\phi+1)}dx_1 \\ &= \frac{1}{\pi}\int_0^{\pi/2}\frac{\lambda_1\lambda_r}{\left(\mu\lambda_2+(1+\gamma_r\csc^2\phi)\lambda_r\right)\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2+(1+\gamma_r\csc^2\phi)\lambda_r\right)}d\phi \end{aligned} \tag{2.24}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect m_1 and m_2. Let $\mathbf{c} = (m_1, m_2, p)$ and $\hat{\mathbf{c}} = (\hat{m}_1, \hat{m}_2, \hat{p})$, be two distinct codewords ($\mathbf{c} \neq \hat{\mathbf{c}}$). Then, the pairwise error probability is given by [15]

$$\begin{aligned} P(\mathbf{c} \to \hat{\mathbf{c}}|\mathbf{h}) &= Q\left(\sqrt{\frac{\|\mathbf{h}\cdot(\mathbf{c}-\hat{\mathbf{c}})\|^2}{2N_0}}\right) \\ &= \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\|\mathbf{h}\cdot(\mathbf{c}-\hat{\mathbf{c}})\|^2}{4N_0\sin^2\phi}\right) d\phi \end{aligned} \tag{2.25}$$

where $Q(x)$ denotes the Gaussian Q-function, $\|\mathbf{h}\cdot(\mathbf{c}-\hat{\mathbf{c}})\|^2 = |h_1|^2|m_1-\hat{m}_1|^2E_s + |h_2|^2|m_2-\hat{m}_2|^2E_s + |h_r|^2|p-\hat{p}|^2E_r$, and the second equation follows from Craig's formula. Without loss of generality assume that $m_1 = 1$, $m_2 = 1$, $p = m_1 \oplus m_2 = 0$ was transmitted. The union bound is used to evaluate a tight upper bound on bit error rate which can be expressed as

$$\begin{aligned} P_{e,1,\mathbf{Z}_5}(\underline{\mathbf{x}}) \leq\, & Q\left(\sqrt{2x_1E_1/N_0 + 2\mu x_2E_2/N_0}\right) + \frac{1}{2}Q\left(\sqrt{2x_1E_1/N_0 + 2x_rE_r/N_0}\right) \\ & + \frac{1}{2}Q\left(\sqrt{2\mu x_2E_2/N_0 + 2x_rE_r/N_0}\right) \end{aligned} \tag{2.26}$$

Averaging over joint distribution of x_1, x_2 and x_r for event $\mathbf{Z}_5$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_5} \leq & \int_0^{\pi/2}\frac{d\phi}{\pi}\int_0^{\infty}\lambda_1 e^{-\lambda_1 x_1(\gamma_1\csc^2\phi+1)}dx_1\int_0^{x_1}\lambda_r e^{-\lambda_r x_r}dx_r\int_0^{x_r}\mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2\csc^2\phi+1)}dx_2 \\ & + \int_0^{\pi/2}\frac{d\phi}{2\pi}\int_0^{\infty}\lambda_1 e^{-\lambda_1 x_1(\gamma_1\csc^2\phi+1)}dx_1\int_0^{x_1}\lambda_r e^{-\lambda_r x_r(\gamma_r\csc^2\phi+1)}dx_r\int_0^{x_r}\mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2 \\ & + \int_0^{\pi/2}\frac{d\phi}{2\pi}\int_0^{\infty}\lambda_1 e^{-\lambda_1 x_1}dx_1\int_0^{x_1} e^{-\lambda_r x_r(\gamma_r\csc^2\phi+1)}dx_r\int_0^{x_r}\mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2\csc^2\phi+1)}dx_2 \\ \leq & \frac{1}{\pi}\int_0^{\pi/2}\left[\frac{\mu\lambda_2}{(1+\gamma_1\csc^2\phi)\left((1+\gamma_1\csc^2\phi)\lambda_1+\lambda_r\right)}\times\right. \\ & \left.\frac{\lambda_r}{\left((1+\gamma_1\csc^2\phi)\lambda_1+\mu\lambda_2(1+\gamma_2\csc^2\phi)+\lambda_r\right)}\right]d\phi \\ & + \frac{1}{2\pi}\int_0^{\pi/2}\left[\frac{\mu\lambda_2}{(1+\gamma_1\csc^2\phi)\left(\lambda_1(1+\gamma_1\csc^2\phi)+\lambda_r(1+\gamma_r\csc^2\phi)\right)}\times\right. \\ & \left.\frac{\lambda_r}{\left(\lambda_1(1+\gamma_1\csc^2\phi)+\mu\lambda_2+\lambda_r(1+\gamma_r\csc^2\phi)\right)}\right]d\phi \\ & + \frac{1}{2\pi}\int_0^{\pi/2}\left[\frac{\mu\lambda_2\lambda_r}{\left(\lambda_1+\lambda_r(1+\gamma_r\csc^2\phi)\right)\left(\lambda_1+\mu\lambda_2(1+\gamma_2\csc^2\phi)+\lambda_r(1+\gamma_r\csc^2\phi)\right)}\right]d\phi \end{aligned} \tag{2.27}$$

where $\gamma_2 = d_2^{-\alpha} E_2/N_0$ is the receive SNR at the destination from S_2. Similarly, since the relay transmits $p = m_1 \oplus m_2$, for the event $\mathbf{Z}_6$, union bound is used to derive $P_{e,1,\mathbf{Z}_6}$.

$$
\begin{aligned}
P_{e,1,\mathbf{Z}_6} \leq & \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} \lambda_r e^{-\lambda_r x_r} dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1 (\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \\
& + \int_0^{\pi/2} \frac{d\phi}{2\pi} \int_0^{\infty} \lambda_r e^{-\lambda_r x_r (\gamma_r \csc^2\phi + 1)} dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1 (\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2} dx_2 \\
& + \int_0^{\pi/2} \frac{d\phi}{2\pi} \int_0^{\infty} \lambda_r e^{-\lambda_r x_r (\gamma_r \csc^2\phi + 1)} dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1} dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \\
& \leq \frac{1}{\pi} \int_0^{\pi/2} \frac{\mu\lambda_1\lambda_2}{\left(\left(1+\gamma_1\csc^2\phi\right)\lambda_1 + \lambda_r\right)\left(\left(1+\gamma_1\csc^2\phi\right)\lambda_1 + \mu\lambda_2\left(1+\gamma_2\csc^2\phi\right) + \lambda_r\right)} d\phi \\
& + \frac{1}{2\pi} \int_0^{\pi/2} \left[\frac{\mu\lambda_1}{\left(1+\gamma_r\csc^2\phi\right)\left(\lambda_1(1+\gamma_1\csc^2\phi) + \lambda_r(1+\gamma_r\csc^2\phi)\right)} \times \right. \\
& \qquad \left. \frac{\lambda_2}{\left(\lambda_1(1+\gamma_1\csc^2\phi) + \mu\lambda_2 + \lambda_r(1+\gamma_r\csc^2\phi)\right)} \right] d\phi \\
& + \frac{1}{2\pi} \int_0^{\pi/2} \left[\frac{\mu\lambda_1}{\left(1+\gamma_r\csc^2\phi\right)\left(\lambda_1 + \left(1+\gamma_r\csc^2\phi\right)\lambda_r\right)} \times \right. \\
& \qquad \left. \frac{\lambda_2}{\left(\lambda_1 + \mu\lambda_2\left(1+\gamma_2\csc^2\phi\right) + \lambda_r\left(1+\gamma_r\csc^2\phi\right)\right)} \right] d\phi
\end{aligned}
\tag{2.28}
$$

Using Equations (2.17), (2.21), (2.22), (2.23), (2.24), (2.27), (2.28), bound on $P_{e,1}$ can be found. This bound is very accurate as shown in simulation results.

Now $P_{e,2,\mathbf{Z}_1}$, $P_{e,2,\mathbf{Z}_2}$, $P_{e,2,\mathbf{Z}_3}$, $P_{e,2,\mathbf{Z}_4}$ will be derived. For events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, the destination decodes m_2 using the signal received directly from S_2. Hence the bit error probability for m_2 for events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$ is given by

$$
P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) = Q\left(\sqrt{2\mu x_2 E_2/N_0}\right) \tag{2.29}
$$

where j = 1, 2, 3, 4. Averaging Eq.(2.29) over x_1, x_2 and x_r yields

$$
P_{e,2,\mathbf{Z}_j} = \iiint\limits_{\mathbf{Z}_j} P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{2.30}
$$

Using limit of integration for $\mathbf{Z}_j$ and using Craig's formula we obtain

$$
\begin{aligned}
P_{e,2,\mathbf{Z}_1} &= \frac{1}{\pi} \int_0^{\pi/2} d\phi \int_0^{\infty} \lambda_1 e^{-\lambda_1 x_1} dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu x_2 \lambda_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \int_0^{x_2} \lambda_r e^{-\lambda_r x_r} dx_r \\
&= \frac{1}{\pi} \int_0^{\pi/2} \frac{\mu\lambda_2\lambda_r}{\left(\lambda_1 + \mu\lambda_2\left(1+\gamma_2\csc^2\phi\right)\right)\left(\lambda_1 + \lambda_r + \mu\lambda_2\left(1+\gamma_2\csc^2\phi\right)\right)} d\phi
\end{aligned}
\tag{2.31}
$$

Using limit of integration for $\mathbf{Z}_2$ and using Craig's formula we obtain

$$\begin{aligned}P_{e,2,\mathbf{Z}_2} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2 \csc^2\phi+1)} dx_2 \int_0^{x_2} \lambda_1 e^{-\lambda_1 x_1} dx_1 \int_0^{x_1} \lambda_r e^{-\lambda_r x_r} dx_r \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{\lambda_1\lambda_r}{(1+\gamma_2\csc^2\phi)(\lambda_1+\mu\lambda_2(1+\gamma_2\csc^2\phi))(\lambda_1+\lambda_r+\mu\lambda_2(1+\gamma_2\csc^2\phi))} d\phi\end{aligned} \tag{2.32}$$

Using limit of integration for $\mathbf{Z}_3$ and using Craig's formula we obtain

$$\begin{aligned}P_{e,2,\mathbf{Z}_3} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} \lambda_r e^{-\lambda_r x_r} dx_r \int_0^{x_r} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2 \csc^2\phi+1)} dx_2 \int_0^{x_2} \lambda_1 e^{-\lambda_1 x_1} dx_1 \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{\mu\lambda_1\lambda_2}{\pi(\lambda_r+\mu\lambda_2(1+\gamma_2\csc^2\phi))(\lambda_1+\lambda_r+\mu\lambda_2(1+\gamma_2\csc^2\phi))} d\phi\end{aligned} \tag{2.33}$$

Using limit of integration for $\mathbf{Z}_4$ and using Craig's formula we obtain

$$\begin{aligned}P_{e,2,\mathbf{Z}_4} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2 \csc^2\phi+1)} dx_2 \int_0^{x_2} \lambda_r e^{-\lambda_r x_r} dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1} dx_1 \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{\lambda_1\lambda_r}{(1+\gamma_2\csc^2\phi)(\lambda_r+\mu\lambda_2(1+\gamma_2\csc^2\phi))(\lambda_1+\lambda_r+\mu\lambda_2(1+\gamma_2\csc^2\phi))} d\phi\end{aligned} \tag{2.34}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect m_1 and m_2, hence we have $P_{e,2,\mathbf{Z}_5} = P_{e,1,\mathbf{Z}_5}$ and $P_{e,2,\mathbf{Z}_6} = P_{e,1,\mathbf{Z}_6}$. Using Equations (2.27), (2.28), (2.17), (2.31), (2.32), (2.33), (2.34), bound on $P_{e,2}$ can be found. This bound is also very accurate as shown in simulation results.

2.4.2 Q function upper bound on Bit Error Probability

The bit error probability for SPNC scheme derived in previous section matches quite well with simulation results. However, it doesn't gives a very good idea about the diversity order and the asymptotic behavior of coding scheme because of non-closed form expressions. This is primarily because of using Craig's formula. In this section we derive bit error probability using bound on Q-function and as shown in simulation results, the bound is fairly tight.

We have the following bound on Q function:

$$Q(x) \leq \frac{1}{2}e^{-\frac{x^2}{2}} \tag{2.35}$$

Using this bound we will re-derive the bit error probability expressions for S_1 and S_2. From Eq.(2.18) we have

$$P_{e,1,\mathbf{Z}_j}(x_1, x_2, x_r) = Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) \tag{2.36}$$

where j = 1, 2, 3, 4. Averaging Eq.(2.36) over x_1, x_2 and x_r yields

$$P_{e,1,\mathbf{Z}_j} = \iiint\limits_{\mathbf{Z}_j} Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{2.37}$$

where $f(\underline{\mathbf{x}}) = \mu\lambda_1\lambda_2\lambda_r e^{-(\lambda_1 x_1 + \mu\lambda_2 \cdot x_2 + \lambda_r x_r)}$. Using limit of integration for $\mathbf{Z}_j$ and Q function bound, Eq.(2.35), we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_1} &\leq \frac{1}{2}\int_0^{\infty} \lambda_1 e^{-x_1\lambda_1(\gamma_1+1)} dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2} dx_2 \int_0^{x_2} \lambda_r e^{-x_r\lambda_r(\gamma_r+1)} dx_r \\ &\leq \frac{1}{2}\frac{\mu\lambda_2\lambda_r}{(1+\gamma_1)\left((1+\gamma_1)\lambda_1 + \mu\lambda_2\right)\left((1+\gamma_1)\lambda_1 + \mu\lambda_2 + (1+\gamma_r)\lambda_r\right)} \end{aligned} \tag{2.38}$$

$$\begin{aligned} P_{e,1,\mathbf{Z}_2} &\leq \frac{1}{2}\int_0^{\infty} \mu\lambda_2 e^{-\mu\lambda_2 x_2} dx_2 \int_0^{x_2} \lambda_1 e^{-x_1\lambda_1(\gamma_1+1)} dx_1 \int_0^{x_1} \lambda_r e^{-x_r\lambda_r(\gamma_r+1)} dx_r \\ &\leq \frac{1}{2}\frac{\lambda_1\lambda_r}{\left((1+\gamma_1)\lambda_1 + \mu\lambda_2\right)\left((1+\gamma_1)\lambda_1 + \mu\lambda_2 + (1+\gamma_r)\lambda_r\right)} \end{aligned} \tag{2.39}$$

$$\begin{aligned} P_{e,1,\mathbf{Z}_3} &\leq \frac{1}{2}\int_0^{\infty} \lambda_r e^{-x_r\lambda_r(\gamma_r+1)} dx_r \int_0^{x_r} \mu\lambda_2 e^{-\mu\lambda_2 x_2} dx_2 \int_0^{x_2} \lambda_1 e^{-x_1\lambda_1(\gamma_1+1)} dx_1 \\ &\leq \frac{1}{2}\frac{\mu\lambda_1\lambda_2}{(1+\gamma_r)\left(\mu\lambda_2 + (1+\gamma_r)\lambda_r\right)\left((1+\gamma_1)\lambda_1 + \mu\lambda_2 + (1+\gamma_r)\lambda_r\right)} \end{aligned} \tag{2.40}$$

$$\begin{aligned} P_{e,1,\mathbf{Z}_4} &\leq \frac{1}{2}\int_0^{\infty} \mu\lambda_2 e^{-\mu\lambda_2 x_2} dx_2 \int_0^{x_2} \lambda_r e^{-x_r\lambda_r(\gamma_r+1)} dx_r \int_0^{x_r} \lambda_1 e^{-x_1\lambda_1(\gamma_1+1)} dx_1 \\ &\leq \frac{1}{2}\frac{\lambda_1\lambda_r}{\left(\mu\lambda_2 + (1+\gamma_r)\lambda_r\right)\left((1+\gamma_1)\lambda_1 + \mu\lambda_2 + (1+\gamma_r)\lambda_r\right)} \end{aligned} \tag{2.41}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect m_1 and m_2. The union bound is used to evaluate a tight upper bound on bit error rate in those cases which can be expressed as

$$\begin{aligned} P_{e,1,\mathbf{Z}_5}(\underline{\mathbf{x}}) \leq\ & Q\left(\sqrt{2x_1 E_1/N_0 + 2\mu x_2 E_2/N_0}\right) + \frac{1}{2}Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) \\ &+ \frac{1}{2}Q\left(\sqrt{2\mu x_2 E_2/N_0 + 2x_r E_r/N_0}\right) \end{aligned} \tag{2.42}$$

Averaging over joint distribution of x_1, x_2 and x_r for event $\mathbf{Z}_5$ and using Q function bound we obtain

$$\begin{aligned}
P_{e,1,\mathbf{Z}_5} \leq & \frac{1}{2}\int_0^\infty \lambda_1 e^{-\lambda_1 x_1(\gamma_1+1)}dx_1 \int_0^{x_1} \lambda_r e^{-\lambda_r x_r}dx_r \int_0^{x_r} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2 \\
& + \frac{1}{4}\int_0^\infty \lambda_1 e^{-\lambda_1 x_1(\gamma_1+1)}dx_1 \int_0^{x_1} \lambda_r e^{-\lambda_r x_r(\gamma_r+1)}dx_r \int_0^{x_r} \mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2 \\
& + \frac{1}{4}\int_0^\infty \lambda_1 e^{-\lambda_1 x_1}dx_1 \int_0^{x_1} e^{-\lambda_r x_r(\gamma_r+1)}dx_r \int_0^{x_r} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2 \\
\leq & \frac{1}{2}\left[\frac{\mu\lambda_2\lambda_r}{(1+\gamma_1)\left((1+\gamma_1)\lambda_1+\lambda_r\right)\left((1+\gamma_1)\lambda_1+\mu\lambda_2(1+\gamma_2)+\lambda_r\right)}\right] \\
& + \frac{1}{4}\left[\frac{\mu\lambda_2\lambda_r}{(1+\gamma_1)\left(\lambda_1(1+\gamma_1)+\lambda_r(1+\gamma_r)\right)\left(\lambda_1(1+\gamma_1)+\mu\lambda_2+\lambda_r(1+\gamma_r)\right)}\right] \\
& + \frac{1}{4}\left[\frac{\mu\lambda_2\lambda_r}{\left(\lambda_1+\lambda_r(1+\gamma_r)\right)\left(\lambda_1+\mu\lambda_2(1+\gamma_2)+\lambda_r(1+\gamma_r)\right)}\right]
\end{aligned} \tag{2.43}$$

Similarly, since the relay transmits $p = m_1 \oplus m_2$, for the event $\mathbf{Z}_6$, union bound is used to derive $P_{e,1,\mathbf{Z}_6}$.

$$\begin{aligned}
P_{e,1,\mathbf{Z}_6} \leq & \frac{1}{2}\int_0^\infty \lambda_r e^{-\lambda_r x_r}dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1(\gamma_1+1)}dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2 \\
& + \frac{1}{4}\int_0^\infty \lambda_r e^{-\lambda_r x_r(\gamma_r+1)}dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1(\gamma_1+1)}dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2}dx_2 \\
& + \frac{1}{4}\int_0^\infty \lambda_r e^{-\lambda_r x_r(\gamma_r+1)}dx_r \int_0^{x_r} \lambda_1 e^{-\lambda_1 x_1}dx_1 \int_0^{x_1} \mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2 \\
\leq & \frac{1}{2}\left[\frac{\mu\lambda_1\lambda_2}{\left((1+\gamma_1)\lambda_1+\lambda_r\right)\left((1+\gamma_1)\lambda_1+\mu\lambda_2(1+\gamma_2)+\lambda_r\right)}\right] \\
& + \frac{1}{4}\left[\frac{\mu\lambda_1\lambda_2}{(1+\gamma_r)\left(\lambda_1(1+\gamma_1)+\lambda_r(1+\gamma_r)\right)\left(\lambda_1(1+\gamma_1)+\mu\lambda_2+\lambda_r(1+\gamma_r)\right)}\right] \\
& + \frac{1}{4}\left[\frac{\mu\lambda_1\lambda_2}{(1+\gamma_r)\left(\lambda_1+(1+\gamma_r)\lambda_r\right)\left(\lambda_1+\mu\lambda_2(1+\gamma_2)+\lambda_r(1+\gamma_r)\right)}\right]
\end{aligned} \tag{2.44}$$

Now $P_{e,2,\mathbf{Z}_1}$, $P_{e,2,\mathbf{Z}_2}$, $P_{e,2,\mathbf{Z}_3}$, $P_{e,2,\mathbf{Z}_4}$ will be derived. For events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, the destination decodes m_2 using the signal received directly from S_2. Hence the bit error probability for m_2 for events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$ is given by

$$P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) = Q\left(\sqrt{2\mu x_2 E_2/N_0}\right) \tag{2.45}$$

where j = 1, 2, 3, 4. Averaging Eq.(2.45) over x_1, x_2 and x_r yields

$$P_{e,2,\mathbf{Z}_j} = \iiint_{\mathbf{Z}_j} P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{2.46}$$

Using limit of integration for $\mathbf{Z}_j$ and using Q function bound we obtain

$$\begin{aligned} P_{e,2,\mathbf{Z}_1} &\leq \frac{1}{2}\int_0^{\infty}\lambda_1 e^{-\lambda_1 x_1}dx_1\int_0^{x_1}\mu\lambda_2 e^{-\mu x_2\lambda_2(\gamma_2+1)}dx_2\int_0^{x_2}\lambda_r e^{-\lambda_r x_r}dx_r \\ &\leq \frac{1}{2}\frac{\mu\lambda_2\lambda_r}{\left(\lambda_1+\mu\lambda_2\left(1+\gamma_2\right)\right)\left(\lambda_1+\lambda_r+\mu\lambda_2\left(1+\gamma_2\right)\right)} \end{aligned} \tag{2.47}$$

$$\begin{aligned} P_{e,2,\mathbf{Z}_2} &\leq \frac{1}{2}\int_0^{\infty}\mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2\int_0^{x_2}\lambda_1 e^{-\lambda_1 x_1}dx_1\int_0^{x_1}\lambda_r e^{-\lambda_r x_r}dx_r \\ &\leq \frac{1}{2}\frac{\lambda_1\lambda_r}{\left(1+\gamma_2\right)\left(\lambda_1+\mu\lambda_2\left(1+\gamma_2\right)\right)\left(\lambda_1+\lambda_r+\mu\lambda_2\left(1+\gamma_2\right)\right)} \end{aligned} \tag{2.48}$$

$$\begin{aligned} P_{e,2,\mathbf{Z}_3} &\leq \frac{1}{2}\int_0^{\infty}\lambda_r e^{-\lambda_r x_r}dx_r\int_0^{x_r}\mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2\int_0^{x_2}\lambda_1 e^{-\lambda_1 x_1}dx_1 \\ &\leq \frac{1}{2}\frac{\mu\lambda_1\lambda_2}{\left(\lambda_r+\mu\lambda_2\left(1+\gamma_2\right)\right)\left(\lambda_1+\lambda_r+\mu\lambda_2\left(1+\gamma_2\right)\right)} \end{aligned} \tag{2.49}$$

$$\begin{aligned} P_{e,2,\mathbf{Z}_4} &\leq \frac{1}{2}\int_0^{\infty}\mu\lambda_2 e^{-\mu\lambda_2 x_2(\gamma_2+1)}dx_2\int_0^{x_2}\lambda_r e^{-\lambda_r x_r}dx_r\int_0^{x_r}\lambda_1 e^{-\lambda_1 x_1}dx_1 \\ &\leq \frac{1}{2}\frac{\lambda_1\lambda_r}{\left(1+\gamma_2\right)\left(\lambda_r+\mu\lambda_2\left(1+\gamma_2\right)\right)\left(\lambda_1+\lambda_r+\mu\lambda_2\left(1+\gamma_2\right)\right)} \end{aligned} \tag{2.50}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect m_1 and m_2, hence we have $P_{e,2,\mathbf{Z}_5} = P_{e,1,\mathbf{Z}_5}$ and $P_{e,2,\mathbf{Z}_6} = P_{e,1,\mathbf{Z}_6}$. Using Equations (2.43), (2.44), (2.17), (2.47), (2.48), (2.49), (2.50), bound on $P_{e,2}$ can be found. This bound is also very tight within a dB as shown in simulation results.

2.4.3 Asymptotic Analysis

In this section, we will derive asymptotic bounds for the bit error probabilities. Assume $\gamma_1 = \gamma_2 = \gamma$, $d_1 = d_2 = d$ and $(d_r/d)^{\alpha} = \delta$. Therefore $\gamma_r = \gamma/\delta$.

At high SNR ($\gamma \to \infty$), collecting dominant terms for $P_{e,1}$ we obtain following bound on $P_{e,1}$:

$$\begin{aligned} P_{e,1} &\leq \frac{1}{2}\frac{\lambda_1\lambda_r}{\left(\left(1+\gamma_1\right)\lambda_1+\mu\lambda_2\right)\left(\left(1+\gamma_1\right)\lambda_1+\mu\lambda_2+\left(1+\gamma_r\right)\lambda_r\right)} \\ &+ \frac{1}{2}\frac{\lambda_1\lambda_r}{\left(\mu\lambda_2+\left(1+\gamma_r\right)\lambda_r\right)\left(\left(1+\gamma_1\right)\lambda_1+\mu\lambda_2+\left(1+\gamma_r\right)\lambda_r\right)} \\ &+ \frac{1}{4}\frac{\mu\lambda_2\lambda_r}{\left(\lambda_1+\lambda_r\left(1+\gamma_r\right)\right)\left(\lambda_1+\mu\lambda_2\left(1+\gamma_2\right)+\lambda_r\left(1+\gamma_r\right)\right)} \\ &+ \frac{1}{2}\frac{\mu\lambda_1\lambda_2}{\left(\left(1+\gamma_1\right)\lambda_1+\lambda_r\right)\left(\left(1+\gamma_1\right)\lambda_1+\mu\lambda_2\left(1+\gamma_2\right)+\lambda_r\right)} \end{aligned} \tag{2.51}$$

Eq. (2.51) can be expressed as:

$$P_{e,1} \leq \frac{1}{2}\frac{\delta}{(\gamma+\mu+1)(2\gamma+\mu+\delta+1)} + \frac{1}{2}\frac{\delta}{(\gamma+\mu+\delta)(2\gamma+\mu+\delta+1)} + \frac{1}{4}\frac{\mu\delta}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} + \frac{1}{2}\frac{\mu}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} \tag{2.52}$$

Clearly for high SNR, the diversity order for S_1 is 2. For $0 < \mu < 1$, and high SNR ($\gamma \to \infty$), the asymptotic bound for bit error probability of S_1 is given by:

$$\begin{aligned} P_{e,1} &\leq \frac{\delta}{2\gamma^2} + \frac{\mu\delta}{4\gamma^2(1+\mu)} + \frac{\mu}{2\gamma^2(1+\mu)} \\ &= \left(\sqrt{\frac{4(1+\mu)}{2\delta+3\mu\delta+2\mu}} \times \gamma\right)^{-2} \end{aligned} \tag{2.53}$$

Now collecting dominant terms of $P_{e,2}$ for high SNR ($\gamma \to \infty$) following is the bound on $P_{e,2}$:

$$\begin{aligned} P_{e,2} \leq &\frac{1}{2}\frac{\mu\lambda_2\lambda_r}{(\lambda_1+\mu\lambda_2(1+\gamma_2))(\lambda_1+\lambda_r+\mu\lambda_2(1+\gamma_2))} \\ &+\frac{1}{2}\frac{\mu\lambda_1\lambda_2}{(\lambda_r+\mu\lambda_2(1+\gamma_2))(\lambda_1+\lambda_r+\mu\lambda_2(1+\gamma_2))} \\ &+\frac{1}{4}\frac{\mu\lambda_2\lambda_r}{(\lambda_1+\lambda_r(1+\gamma_r))(\lambda_1+\mu\lambda_2(1+\gamma_2)+\lambda_r(1+\gamma_r))} \\ &+\frac{1}{2}\frac{\mu\lambda_1\lambda_2}{((1+\gamma_1)\lambda_1+\lambda_r)((1+\gamma_1)\lambda_1+\mu\lambda_2(1+\gamma_2)+\lambda_r)} \end{aligned} \tag{2.54}$$

Eq.(2.54) can be expressed as:

$$P_{e,2} \leq \frac{1}{2}\frac{\mu\delta}{(1+\mu\gamma)(1+\delta+\mu\gamma)} + \frac{1}{2}\frac{\mu}{(\delta+\mu\gamma)(1+\delta+\mu\gamma)} + \frac{1}{4}\frac{\mu\delta}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} + \frac{1}{2}\frac{\mu}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} \tag{2.55}$$

Clearly for high SNR, the diversity order for S_2 is 2. For $0 < \mu < 1$, and high SNR ($\gamma \to \infty$), the asymptotic bound for bit error probability of S_2 is given by:

$$\begin{aligned} P_{e,2} &\leq \frac{\delta}{2\mu\gamma^2} + \frac{1}{2\mu\gamma^2} + \frac{\mu\delta}{4\gamma^2(1+\mu)} + \frac{\mu}{2\gamma^2(1+\mu)} \\ &= \left(\sqrt{\frac{4\mu(1+\mu)}{2\mu^2+\mu^2\delta+2\mu\delta+2\mu+2\delta+2}} \times \gamma\right)^{-2} \end{aligned} \tag{2.56}$$

We will now derive approximate bit error probability for traditional network coding. When the relay performs traditional network coding, then it always transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect m_1 and m_2. In this case the bit error probabilities for S_1 and S_2 are equal. The union bound is used to evaluate a tight upper bound on bit error rate which can be expressed as:

$$P_{NC}(\underline{\mathbf{x}}) \leq Q\left(\sqrt{2x_1E_1/N_0 + 2\mu x_2E_2/N_0}\right) + \frac{1}{2}Q\left(\sqrt{2x_1E_1/N_0 + 2x_rE_r/N_0}\right) + \frac{1}{2}Q\left(\sqrt{2\mu x_2E_2/N_0 + 2x_rE_r/N_0}\right) \tag{2.57}$$

Using Q function approximation, Eq.(2.35), and averaging over joint distribution of x_1, x_2 and x_r we obtain:

$$P_{NC} \leq \frac{1}{2\gamma_1\gamma_2} + \frac{1}{4\gamma_1\gamma_r} + \frac{1}{4\gamma_2\gamma_r} \tag{2.58}$$

For the case when $\gamma_1 = \gamma_2 = \gamma$, $d_1 = d_2 = d$ and $(d_r/d)^\alpha = \delta$, the bit error probability for network coding can be expressed as:

$$P_{NC} \leq \left(\sqrt{\frac{2}{\delta+1}} \times \gamma\right)^{-2} \tag{2.59}$$

For $\mu = 0.1$, $d_r/d = 0.5$ i.e $\delta = 0.0625$, we have

$$\begin{aligned} P_{e,1} &\leq (3.5777 \times \gamma)^{-2} \\ P_{e,2} &\leq (0.4320 \times \gamma)^{-2} \\ P_{NC} &\leq (1.3720 \times \gamma)^{-2} \end{aligned} \tag{2.60}$$

Hence the coding gain of S_1 over $S_2 \approx 10\times \log_{10}(3.5777/0.4320) = 9.18$ dB. The coding gain of S_1 for SPNC scheme compared to traditional network coding is $\approx 10\times \log_{10}(3.5777/1.3720)$ $= 4.16$ dB whereas the loss for S_2 is $\approx 10\times \log_{10}(1.3720/0.4320) = 5.02$ dB.

2.4.4 Effect of Channel Estimation Error

In practice, the channel estimation may not be perfect. The estimated channel fading gains can be modeled as [18]

$$\hat{h}_i = h_i - e_i, \quad \text{i} = 1,\ 2,\ \text{r} \tag{2.61}$$

where e_i is distributed as $\mathcal{CN}(0, \sigma^2_{e,i})$ and represents the channel estimation error. For pilot symbol aided minimum mean squared error (MMSE) channel estimation, the error variance is given by [17]

$$\sigma^2_{e,i} = \frac{1}{1 + \frac{\pi}{L\omega_{ND}} \cdot \frac{E_{ps,i}}{N_0}} \tag{2.62}$$

where L is the rate of insertion of pilot symbols, $E_{ps,i}$ is the average received energy per pilot symbol from the i^{th} source at the destination and ω_{ND} is the normalized Doppler frequency, normalized with respect to sampling frequency. Then, the effective receive SNR is given by

$$\gamma_{e,i} = \frac{E_i(d_i^{-\alpha} - \sigma_{e,i}^2)}{N_0 + E_i\sigma_{e,i}^2} \tag{2.63}$$

which replaces γ_i in Equations (2.21), (2.22), (2.23), (2.24), (2.27), (2.28) for S_1 when the channel estimation is not perfect and in Equations (2.27), (2.28), (2.31), (2.32), (2.33), (2.34) for S_2.

2.4.5 Outage analysis

In following subsections we will analyze the outage probability of the proposed scheme and present the diversity analysis for various scenarios. We consider a point to point communication system as the base line communication system for comparison. The received signal at the destination for a point to point communication is given by

$$y = h\mathcal{X}(m) + n \tag{2.64}$$

where $\mathcal{X}(m)$ is Gaussian distributed with average energy E_s. The instantaneous signal to noise ratio (SNR) of the channel is given by $|h|^2E_s/N_0$, $|h|^2$ is exponentially distributed. When the instantaneous SNR is less than certain threshold, the source is said to be in *outage*. The outage probability is given by:

$$P_o(R) = P\left(|h|^2 < \frac{2^R - 1}{d^{-\alpha}E_s/N_0}\right) = 1 - e^{\left(-\frac{2^R-1}{d^{-\alpha}E_s/N_0}\right)} \tag{2.65}$$

where R is the spectral efficiency of the system in bits per channel use.

2.4.6 Outage Analysis for two user, one relay scenario

We assume that each source transmits information at the rate of R bits per channel use. Let $g_1 = |h_1|^2$, $g_2 = |h_2|^2/\mu$ and $g_{r_1} = |h_{r_1}|^2$ denote the channel power gains. These channel gains are independent and hence the joint probability density function (pdf) of $\mathbf{g} = (g_1, g_2, g_{r_1})$ is a product of their marginal pdf i.e. $f(\mathbf{g}) = \mu d_{1d}^{\alpha} d_{2d}^{\alpha} d_{r_1d}^{\alpha} e^{-(d_{1d}^{\alpha}g_1 + \mu d_{2d}^{\alpha}g_2 + d_{r_1d}^{\alpha}g_{r_1})}$.

Let $P_{o,ir_1} = 1 - \exp\left(-d_{ir_1}^{\alpha}\Gamma\right)$ denote the probability that S_i is in outage at the relay where

$$\Gamma = \frac{2^{(3R/2)} - 1}{E_s/N_0} \tag{2.66}$$

The factor 3/2 is to account for the fact that three channel uses are performed in transmitting two message packets [21]. There are 4 possible disjoint events depending on whether relay is able to decode $\mathbf{m}_1$ and $\mathbf{m}_2$ successfully or not.

1. **Case I:** Consider the case in which both $\mathbf{m}_1$ and $\mathbf{m}_2$ are decoded successfully at the relay. Let $\mathcal{G}$ denote the event $g_2 > \min\{g_{r_1}, g_1\}$ and $\mathcal{G}^c$ denote the event $g_2 < \min\{g_{r_1}, g_1\}$. Under the event $\mathcal{G}$, $\mathbf{m}_{r_1} = \mathbf{m}_1$ is transmitted by the relay and the destination combines $\mathbf{y}_1$ and $\mathbf{y}_{r_1}$ using the maximal ratio combining (MRC) rule to decode $\mathbf{m}_1$, while $\mathbf{m}_2$ is decoded based on $\mathbf{y}_2$. The events $\mathcal{G}$ and $\mathcal{G}^c$ can be divided into the following disjoint events.

Event	Channel Condition	Event	Channel Condition
$\mathcal{G}_1$	$g_1 > g_2 > g_{r_1}$	$\mathcal{G}_2$	$g_2 > g_1 > g_{r_1}$
$\mathcal{G}_3$	$g_{r_1} > g_2 > g_1$	$\mathcal{G}_4$	$g_2 > g_{r_1} > g_1$
$\mathcal{G}_1^c$	$g_1 > g_{r_1} > g_2$	$\mathcal{G}_2^c$	$g_{r_1} > g_1 > g_2$

Table 2.1 Disjoint events for outage probability for SPNC

The outage probability for $\mathbf{m}_1$ and $\mathbf{m}_2$ under the event $\mathcal{G}$ can be expressed as

$$\begin{aligned} P_{o,1,I,\mathcal{G}} &= \sum_{j=1}^{4} Pr\left(\{g_1 + g_{r_1} < \Gamma\} \cap \mathcal{G}_j\right) \\ P_{o,2,I,\mathcal{G}} &= \sum_{j=1}^{4} Pr\left(\{\mu g_2 < \Gamma\} \cap \mathcal{G}_j\right) \end{aligned} \tag{2.67}$$

where $Pr\left(\{g_1 + g_{r_1} < \Gamma\} \cap \mathcal{G}_j\right)$ can be calculated by integration of $f(\mathbf{g})$ over the volume defined by the events $\{g_1 + g_{r_1} < \Gamma\}$ and $\mathcal{G}_j$. $Pr\left(\{\mu g_2 < \Gamma\} \cap \mathcal{G}_j\right)$ can be calculated similarly.

Under the event $\mathcal{G}^c$ ($\mathbf{m}_{r_1} = \mathbf{m}_1 \oplus \mathbf{m}_2$), $\mathbf{m}_1$ is in outage when both S_1-D link and R-D link are in outage or both S_1-D link and S_2-D link are in outage. Similarly, $\mathbf{m}_2$ is in outage when both S_2-D link and R-D link are in outage or both S_2-D link and S_1-D link are in outage. Hence, the outage probability of $\mathbf{m}_1$ and $\mathbf{m}_2$ under the event $\mathcal{G}^c$ can be expressed as

$$P_{o,1,I,\mathcal{G}^c} = \sum_{j=1}^{2} Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G}_j^c) + Pr\left(\{g_1 < \Gamma\} \cap \{\mu g_2 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}_j^c\right)$$
$$P_{o,2,I,\mathcal{G}^c} = \sum_{j=1}^{2} Pr(\{\mu g_2 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G}_j^c) + Pr\left(\{\mu g_2 < \Gamma\} \cap \{g_1 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}_j^c\right) \tag{2.68}$$

Since $\mathcal{G}_i$'s and $\mathcal{G}_i^c$'s are mutually disjoint events, averaging (2.67) and (2.68) over channel power gains and summing them yields a closed form expression, Eq.(2.69), for the average outage probability of $\mathbf{m}_1$ and $\mathbf{m}_2$ for the case I.

$$P_{o,1,I} = \frac{1+\delta+2\mu-\delta\mu+\mu^2}{(1+\mu)(1+\delta+\mu)} + \frac{4\delta\mu e^{-\frac{1}{2}\Gamma(1+\delta+\mu)}}{(1+\delta+\mu)(1-\delta+\mu)(1-\delta-\mu)} + \frac{\mu e^{-\Gamma(1+\delta+\mu)}}{(\delta+\mu)(1+\delta+\mu)} + \frac{2\delta e^{-\Gamma(1+\mu)}}{(1+\mu)(1-\delta+\mu)} - \frac{e^{-\Gamma\delta}}{1-\delta+\mu} + \frac{e^{-\Gamma}(\mu^2-\delta^2-\mu)}{(\delta+\mu)(1-\delta-\mu)}$$
$$P_{o,2,I} = 1 - e^{-\Gamma} - e^{-\Gamma(1+\delta)} + \frac{1+\delta}{1+\delta+\mu} e^{-\Gamma(1+\delta+\mu)} + \frac{\mu}{1+\delta+\mu} e^{-\frac{\Gamma(1+\delta+\mu)}{\mu}} \tag{2.69}$$

where it is assumed that $d_{1d} = d_{2d} = d = 1$ and $\delta \triangleq (d_{r_1d})^\alpha$.

2. **Case II:** Consider the case in which $\mathbf{m}_1$ is decoded successfully at the relay while $\mathbf{m}_2$ is not and hence $\mathbf{m}_{r_1} = \mathbf{m}_1$. In this case, the destination combines $\mathbf{y}_1$ and $\mathbf{y}_{r_1}$ using the MRC rule to decode $\mathbf{m}_1$ and decodes $\mathbf{m}_2$ based on $\mathbf{y}_2$ only. Hence the outage probabilities for $\mathbf{m}_1$ and $\mathbf{m}_2$ can be expressed as:

$$P_{o,1,II} = Pr\left(g_1 + g_{r_1} < \Gamma\right) = \left(1 - \frac{1}{1-\delta} e^{-\delta\Gamma} + \frac{\delta}{1-\delta} e^{-\Gamma}\right)$$
$$P_{o,2,II} = Pr\left(\mu g_2 < \Gamma\right) = 1 - e^{-\Gamma} \tag{2.70}$$

3. **Case III:** Consider the case in which $\mathbf{m}_2$ is decoded successfully at the relay while $\mathbf{m}_1$ is not and hence $\mathbf{m}_{r_1} = \mathbf{m}_2$. In this case, the destination combines $\mathbf{y}_2$ and $\mathbf{y}_{r_1}$ using the MRC rule to decode $\mathbf{m}_2$ and decodes $\mathbf{m}_1$ based on $\mathbf{y}_1$ only. Hence the outage probabilities for $\mathbf{m}_1$ and $\mathbf{m}_2$ can be expressed as:

$$P_{o,1,III} = Pr\left(g_1 < \Gamma\right) = 1 - e^{-\Gamma}$$
$$P_{o,2,III} = Pr\left(\mu g_2 + g_{r_1} < \Gamma\right) = \left(1 - \frac{1}{1-\delta} e^{-\delta\Gamma} + \frac{\delta}{1-\delta} e^{-\Gamma}\right) \tag{2.71}$$

4. **Case IV:** Consider the case in which both $\mathbf{m}_1$ and $\mathbf{m}_2$ are decoded unsuccessfully at the relay and hence the relay remains silent. The destination decodes $\mathbf{m}_1$ based on $\mathbf{y}_1$ only and $\mathbf{m}_2$ based on $\mathbf{y}_2$ only. The outage probabilities for $\mathbf{m}_1$ and $\mathbf{m}_2$ are given by

$$\begin{aligned} P_{o,1,IV} &= Pr\left(g_1 < \Gamma\right) = 1 - e^{-\Gamma} \\ P_{o,2,IV} &= Pr\left(\mu g_2 < \Gamma\right) = 1 - e^{-\Gamma} \end{aligned} \tag{2.72}$$

The average outage probability of $\mathbf{m}_1$ and $\mathbf{m}_2$ can be obtained in a closed form expression by taking a weighted sum of Eqs.(2.69), (2.70), (2.71) and (2.72), weighted by the probability of each respective case as

$$\begin{aligned} P_{o,1} =& e^{-\beta\Gamma} e^{-\beta\Gamma} P_{o,1,I} + e^{-\beta\Gamma}\left(1 - e^{-\beta\Gamma}\right) P_{o,1,II} + \left(1 - e^{-\beta\Gamma}\right) e^{-\beta\Gamma} P_{o,1,III} \\ &+ (1 - e^{-\beta\Gamma})(1 - e^{-\beta\Gamma}) P_{o,1,IV} \end{aligned} \tag{2.73}$$

$$\begin{aligned} P_{o,2} =& e^{-\beta\Gamma} e^{-\beta\Gamma} P_{o,2,I} + \left(1 - e^{-\beta\Gamma}\right) e^{-\beta\Gamma} P_{o,2,II} + e^{-\beta\Gamma}\left(1 - e^{-\beta\Gamma}\right) P_{o,2,III} \\ &+ (1 - e^{-\beta\Gamma})(1 - e^{-\beta\Gamma}) P_{o,2,IV} \end{aligned} \tag{2.74}$$

where where it is assumed that $d_{1r_1} = d_{2r_1} = d_{sr_1}$, $\beta \triangleq d_{sr_1}^{\alpha}$. As shown by simulations, the theoretical analysis matches very closely with simulation. At high SNR, $E_s/N_0 \gg 1$, the probability of occurrence of Case II, III and IV is negligible. Using the second order approximation for $e^{-x} \approx 1 - x + x^2/2$ yields the following asymptotic approximation for the outage probabilities:

$$P_{o,1} \approx \frac{\Gamma^2(\mu + \delta + 2\beta)}{2} \triangleq \tilde{P}_{o,1} \tag{2.75}$$

$$P_{o,2} \approx \frac{\Gamma^2((1+\delta)(1+\mu^2) + 2\mu\beta)}{2\mu} \triangleq \tilde{P}_{o,2} \tag{2.76}$$

The high SNR analysis clearly shows that second order diversity can be provided for both prioritized and non-prioritized nodes. The simulation results show that these approximations are quite accurate for outage probability below 10^{-2}.

2.4.7 Outage Analysis for Cluster based Grouping

In this subsection, we will discuss the outage probability for cluster based grouping. The SPNC coding rule for grouping is given by Eq.(2.15). The pdf of channel strength of group

1 head is given by $f_{G_1}(g_1) = \lambda_1 e^{-\lambda_1 g_1}$ where $\lambda_1 = d_1^{\alpha}$, d_1 denotes the distance of group head 1 from the destination. The pdf of channel strength of group 2 head is given by $f_{G_2}(g_2) = \lambda_2 e^{-\lambda_2 g_2}$ where $\lambda_2 = d_2^{\alpha}$. The pdf of channel strength of relay 1 is given by $f_{G_{r_1}}(g_{r_1}) = \lambda_{r_1}$ $e^{-\lambda_{r_1} g_{r_1}}$ where $\lambda_{r_1} = d_{r_1}^{\alpha}$. Since G_1, G_2 and g_{r_1} are independent, hence their joint pdf is the product of respective marginal pdfs i.e. $f(G_1, G_2, G_{r_1}) = \lambda_1 \lambda_2 \lambda_{r_1} e^{-\lambda_1 g_1 - \lambda_2 g_2 - \lambda_{r_1} g_{r_1}}$

The outage probability for Group head 1 to destination link is given by

$$P_o = Pr\left(G_1 < \frac{2^{\frac{N*R}{2}} - 1}{E_s/N_0}\right) \tag{2.77}$$

where

$$E_s = \frac{E_b * N * R}{(N+1)} \tag{2.78}$$

and E_b denotes energy per information bit and total $N+1$ channel uses are performed to transmit N information messages. Similarly, the outage probability for group head 2 to destination link and the relay can be calculated.

- Case 1: When the group heads are located at approximately same distance from the relay and destination, then the outage analysis as given in the previous section for two sources, one relay can be applied.
- Case 2: When the group heads are located at different distances from the relay and destination, then the outage analysis can be performed by analyzing disjoint events. For mid to high SNR range, the relay will be able to successfully decode the messages both the group heads. We will denote this event as $\mathcal{E}$. The probability of occurrence of the event $\mathcal{E}$ is given by

$$P(\mathcal{E}) = e^{-\lambda_{1r_1}\Gamma} e^{-\lambda_{2r_1}\Gamma} \tag{2.79}$$

where $\lambda_{1r_1} = d_{1r_1}^{\alpha}$, $\lambda_{2r_1} = d_{2r_1}^{\alpha}$, d_{1r_1} denotes the distance between group head 1 and the relay R_1, d_{2r_1} denotes the distance between group head 2 and the relay. Also Γ can be calculated as

$$\Gamma = \frac{2^{N*R/2} - 1}{E_s/N_0} \tag{2.80}$$

Let $\mathcal{G}$ denotes the event $G_2 > \mu \cdot \min\{G_{r_1}, G_1\}$ and $\mathcal{G}^c$ denotes the event $G_2 < \mu \cdot \min\{G_{r_1}, G_1\}$. Since these two events are disjoint, therefore the outage probability of groups can be calculated for these individual events and then summed up to give total outage probability.

$$P_{o,1}(\mathcal{E}) = P_{o,1}(\mathcal{G}) \cdot P(\mathcal{E}) + P_{o,1}(\mathcal{G}^c) \cdot P(\mathcal{E})$$
$$P_{o,2}(\mathcal{E}) = P_{o,2}(\mathcal{G}) \cdot P(\mathcal{E}) + P_{o,2}(\mathcal{G}^c) \cdot P(\mathcal{E}) \quad (2.81)$$

The events $\mathcal{G}$ and $\mathcal{G}^c$ can be divided into the following disjoint events.

Event	Channel Condition	Event	Channel Condition
$\mathcal{G}_1$	$G_1 > G_2 > G_{r_1}$	$\mathcal{G}_2$	$G_2 > G_1 > G_{r_1}$
$\mathcal{G}_3$	$G_{r_1} > G_2 > G_1$	$\mathcal{G}_4$	$G_2 > G_{r_1} > G_1$
$\mathcal{G}_1^c$	$G_1 > G_{r_1} > G_2$	$\mathcal{G}_2^c$	$G_{r_1} > G_1 > G_2$

Table 2.2 Disjoint events for outage probability for SPNC with cluster based grouping.

Under the event $\mathcal{G}$, $\mathbf{M}_{r_1} = \mathbf{M}_1$ is transmitted by the relay. The destination decodes message packet of group 1 by performing maximal ratio combining(MRC) of the packets received directly from the group 1 head and the packet from relay, $\mathbf{y}_{r_1}$. Therefore the outage probability of group 1 under the event $\mathcal{G}$ is given by

$$P_{o,1}(\mathcal{G}) = Pr(\{G_1 + g_{r_1} < \Gamma\} \cap \mathcal{G}) \quad (2.82)$$

Using the joint pdf, $f(G_1, G_2, G_{r_1})$, the above probability can be calculated as

$$P_{o,1}(\mathcal{G}) = \sum_{j=1}^{4} \iiint_{V_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} \quad (2.83)$$

where V_j is the volume defined by $(\{G_1 + G_{r_1} < \Gamma\} \cap \mathcal{G}_j)$. The above probability can be upper bounded as

$$P_{o,1}(\mathcal{G}) \leq Pr(\{G_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G}) \quad (2.84)$$

The upper bound probability can be calculated as

$$P_{o,1}(\mathcal{G}) = \sum_{j=1}^{4} \iiint_{V'_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} \quad (2.85)$$

where V'_j is the volume defined by $(\{G_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G})$. Under the event $\mathcal{G}$, the message packet from group 2 head is decoded via direct link. Hence the outage probability of group 2 under the event $\mathcal{G}$ is given by

$$P_{o,2}(\mathcal{G}) = Pr(\{G_2 < \Gamma\} \cap \mathcal{G}) \quad (2.86)$$

Using the joint pdf, $f(G_1, G_2, G_{r_1})$, the above probability can be calculated as

$$P_{o,2}(\mathcal{G}) = \sum_{j=1}^{4} \iiint_{U_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} \quad (2.87)$$

where U_j is the volume defined by $(g_2 < \Gamma \cap \mathcal{G}_j)$. Under the event $\mathcal{G}^c$, $\mathbf{M}_{r_1} = \mathbf{M}_1 \oplus \mathbf{M}_2$ is transmitted by the relay. The destination decodes the source packets jointly using direct transmissions and the network coded packet from the relay. The destination will not be able to decode packet received from group 1 head if any of the following events occur

1. $G_1 < \Gamma$ and $G_{r_1} < \Gamma$
2. $G_1 < \Gamma$ and $G_2 < \Gamma$ and $G_{r_1} > \Gamma$

Therefore the probability of outage of group 1 for the event $\mathcal{G}^c$ is given by

$$P_{o,1}(\mathcal{G}^c) = Pr(G_1 < \Gamma \cap G_{r_1} < \Gamma \cap \mathcal{G}^c) + Pr(G_1 < \Gamma \cap G_2 < \Gamma \cap G_{r_1} > \Gamma \cap \mathcal{G}^c) \quad (2.88)$$

$$P_{o,1}(\mathcal{G}^c) = \sum_{j=1}^{2} \iiint_{W_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} + \iiint_{X_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} \quad (2.89)$$

where W_j is the volume defined by $(G_1 < \Gamma \cap G_{r_1} < \Gamma \cap \mathcal{G}_i^c)$ and X_j is the volume defined by $(G_1 < \Gamma \cap G_2 < \Gamma \cap G_{r_1} > \Gamma \cap \mathcal{G}_i^c)$. The outage probability for group 2 for the event $\mathcal{G}^c$ can be calculated similarly and is given by

$$P_{o,2}(\mathcal{G}^c) = Pr(G_2 < \Gamma \cap G_{r_1} < \Gamma \cap \mathcal{G}^c) + Pr(G_2 < \Gamma \cap G_1 < \Gamma \cap G_{r_1} > \Gamma \cap \mathcal{G}^c) \quad (2.90)$$

$$P_{o,2}(\mathcal{G}^c) = \sum_{j=1}^{2} \iiint_{Y_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} + \iiint_{Z_j} f(G_1, G_2, G_{r_1}) dg_1 dg_2 dg_{r_1} \quad (2.91)$$

where Y_j is the volume defined by $(G_2 < \Gamma \cap G_{r_1} < \Gamma \cap \mathcal{G}_i^c)$ and Z_j is the volume defined by $(G_2 < \Gamma \cap G_1 < \Gamma \cap G_{r_1} > \Gamma \cap \mathcal{G}_i^c)$.

The exact outage probability for group 1 under the event $\mathcal{E}$ can be calculated by numerical integration of Eq.(2.83), (2.89). By integrating Eq. (2.85) and (2.89) and simplification, the upper bound for group 1 can be calculated as

$$P_{o,1}(\mathcal{E}) \leq \Bigg(\left(1 - e^{-\lambda_1 \Gamma}\right)\left(1 - e^{-\lambda_{r_1} \Gamma}\right) + e^{-\lambda_{r_1}\Gamma} \left(1 - e^{-\lambda_1 \Gamma} - \frac{\lambda_1}{\lambda_1 + \mu\lambda_2}\left(1 - e^{-\Gamma(\lambda_1 + \mu\lambda_2)}\right)\right)\Bigg) P(\mathcal{E}) \quad (2.92)$$

Similarly, the exact outage probability for group 2 under the event $\mathcal{E}$ can be calculated by integration of Eq.(2.87), (2.91). After integrating and simplification, the outage probabilities of the group 2 can be derived as

$$P_{o,2}(\mathcal{E})=\Bigg(1-e^{-\lambda_2\Gamma}-\frac{\mu\lambda_2}{\lambda_1+\mu\lambda_2+\lambda_{r_1}}\left(1-e^{-\frac{\Gamma(\lambda_1+\mu\lambda_2+\lambda_{r_1})}{\mu}}\right)$$
$$+\frac{\mu\lambda_2}{\lambda_1+\mu\lambda_2+\lambda_{r_1}}\left(\lambda_{r_1}e^{-\Gamma(\lambda_1+\mu\lambda_2+\lambda_{r_1})}+(\lambda_1+\mu\lambda_2)(1-e^{-\lambda_{r_1}\Gamma})-\lambda_{r_1}e^{-\lambda_{r_1}\Gamma}\right)$$
$$+e^{-\lambda_{r_1}\Gamma}\left(1-e^{-\lambda_1\Gamma}-\frac{\lambda_1}{\lambda_1+\mu\lambda_2}\left(1-e^{-\Gamma(\lambda_1+\mu\lambda_2)}\right)\right)\Bigg)P(\mathcal{E}) \tag{2.93}$$

In the event $\mathcal{E}^c$ i.e. when atleast one of the group head is not decoded successfully at the relay, then the relay forwards the message of the group head which has been decoded successfully. If both the group heads are not decoded successfully, then the relay remains silent. Let $P_{o,1,r_1}=1-e^{-\lambda_{1r_1}}$ denote the outage probability of group 1 head at the relay and $P_{o,2,r_1}=1-e^{-\lambda_{2r_1}}$ denote the outage probability of group 2 head at the relay.

1. **Case I:** Consider the case in which $\mathbf{M}_1$ is decoded successfully at the relay while $\mathbf{M}_2$ is not and hence $\mathbf{M}_{r_1}=\mathbf{M}_1$. In this case, the destination combines messages received from the group 1 head and relay using MRC and decodes $\mathbf{M}_1$. The destination decodes message from group 2 head, $\mathbf{M}_2$, based on direct transmission from group 2 head. Hence the outage probabilities for $\mathbf{M}_1$ and $\mathbf{M}_2$ can be expressed as:

$$P_{o,1,I}(\mathcal{E}^c)=Pr\left(G_1+G_{r_1}<\Gamma\right)(1-P_{o,1,r_1})P_{o,2,r_1}$$
$$=\left(1-\frac{\lambda_1}{\lambda_1-\lambda_{r_1}}e^{-\lambda_{r_1}\Gamma}+\frac{\lambda_{r_1}}{\lambda_1-\lambda_{r_1}}e^{-\lambda_1\Gamma}\right)e^{-\lambda_{1r_1}\Gamma}(1-e^{-\lambda_{2r_1}\Gamma}) \tag{2.94}$$
$$P_{o,2,I}(\mathcal{E}^c)=Pr\left(G_2<\Gamma\right)(1-P_{o,1,r_1})P_{o,2,r_1}$$
$$=\left(1-e^{-\lambda_2\Gamma}\right)e^{-\lambda_{1r_1}\Gamma}(1-e^{-\lambda_{2r_1}\Gamma}) \tag{2.95}$$

2. **Case II:** Consider the case in which $\mathbf{M}_2$ is decoded successfully at the relay while $\mathbf{M}_1$ is not and hence $\mathbf{M}_{r_1}=\mathbf{M}_2$. In this case, the destination combines messages received from the group 2 head and relay using MRC and decodes $\mathbf{M}_2$. The destination decodes message from group 1 head, $\mathbf{M}_1$, based on direct transmission from group 1 head. Hence the outage probabilities for $\mathbf{M}_1$ and $\mathbf{M}_2$ can be expressed as:

$$P_{o,1,II}(\mathcal{E}^c)=Pr\left(G_1<\Gamma\right)P_{o,1,r_1}(1-P_{o,2,r_1})$$

$$= \left(1 - e^{-\lambda_1 \Gamma}\right)(1 - e^{-\lambda_{1r_1}\Gamma})e^{-\lambda_{2r_1}\Gamma} \tag{2.96}$$

$$P_{o,2,II}(\mathcal{E}^c) = Pr\left(G_2 + G_{r_1} < \Gamma\right) P_{o,1,r_1}(1 - P_{o,2,r_1})$$
$$= \left(1 - \frac{\lambda_2}{\lambda_2 - \lambda_{r_1}} e^{-\lambda_{r_1}\Gamma} + \frac{\lambda_{r_1}}{\lambda_2 - \lambda_{r_1}} e^{-\lambda_2 \Gamma}\right)(1 - e^{-\lambda_{1r_1}\Gamma})e^{-\lambda_{2r_1}\Gamma} \tag{2.97}$$

3. **Case III:** Consider the case in which both group heads are decoded unsuccessfully at the relay. Then the relay remains silent and the destination decodes messages from group heads via direct transmission only. Hence the outage probabilities for the groups can expressed as

$$P_{o,1,III}(\mathcal{E}^c) = Pr\left(G_1 < \Gamma\right)(1 - P_{o,1,r_1})(1 - P_{o,2,r_1})$$
$$= \left(1 - e^{-\lambda_1 \Gamma}\right)(1 - e^{-\lambda_{1r_1}\Gamma})(1 - e^{-\lambda_{2r_1}\Gamma}) \tag{2.98}$$

$$P_{o,2,III}(\mathcal{E}^c) = Pr\left(G_2 < \Gamma\right)(1 - P_{o,1,r_1})(1 - P_{o,2,r_1})$$
$$= \left(1 - e^{-\lambda_2 \Gamma}\right)(1 - e^{-\lambda_{1r_1}\Gamma})(1 - e^{-\lambda_{2r_1}\Gamma}) \tag{2.99}$$

Using Eqs. (2.92),(2.94), (2.96) and (2.98) the upper bound in closed form for the total outage probability of group 1 can be calculated as

$$P_{o,1} \leq P_{o,1}(\mathcal{E}) + P_{o,1,I}(\mathcal{E}^c) + P_{o,1,II}(\mathcal{E}^c) + P_{o,1,III}(\mathcal{E}^c) \tag{2.100}$$

Using Eqs. (2.93),(2.95), (2.97) and (2.99), the exact outage probability for group 2 can be calculated in closed form as

$$P_{o,2} = P_{o,2}(\mathcal{E}) + P_{o,2,I}(\mathcal{E}^c) + P_{o,2,II}(\mathcal{E}^c) + P_{o,2,III}(\mathcal{E}^c) \tag{2.101}$$

At high SNR, $E_s/N_0 \gg 1$, the second order approximation $e^{-x} \approx 1 - x + x^2/2$ yields the following asymptotic approximation for the outage probabilities:

$$P_{o,1} \approx \Gamma^2 \left(\frac{\lambda_1(2\lambda_{r_1} + \mu\lambda_2 + 2\lambda_{1r_1})}{2}\right) \tag{2.102}$$

$$P_{o,2} \approx \Gamma^2 \left(\frac{\lambda_2(\lambda_1 + \lambda_{r_1} + 2\mu\lambda_{2r_1} + \mu^2\lambda_1 + \mu^2\lambda_1\lambda_{r_1})}{2\mu}\right) \tag{2.103}$$

Here, in order to derive asymptotic result for group 1, we have used the upper bound of Eq.(2.92) and exact outage probabilities of Eqs.(2.94), (2.96) and (2.98). For deriving asymptotic result for group 2, we have used exact outage probabilities of Eqs.

(2.93),(2.95), (2.97) and (2.99). Clearly our proposed scheme achieves maximal diversity order of 2.

2.4.8 Outage Analysis for Normal Grouping

In this subsection, we will discuss the derivation of outage probability for SPNC with grouping. In general for N sources and one relay, there will be 2^N disjoint events to be analyzed for outage probability. The exact outage analysis for 2^N events is prohibitively complex and yields intractable results. For mid to high SNR range, only one out of 2^N events dominates the outage probability. This is the event when relay is able to successfully decode all the sources and hence all the sources are considered for SPNC. We will denote this event by $\mathcal{E}$. This is shown by simulation result in Fig. 2.4. As shown by the simulation result, all sources are decoded successfully at the relay with very high probability even when number of sources are as high as 20. For simulation purpose, we considered the relay to be at normalized distance of 0.5 from all the sources and the sources transmitting at spectral efficiency of 0.5 bits per channel use.

For the derivation of outage probability we will assume that the N users are divided into 2 groups. The SPNC coding rule for grouping is given by Eq.(2.16). Without loss of generality we assume that group 1 is the prioritized group with $N_1 > 1$ users and group 2 is non prioritized group with $N_2 > 1$ users and $N_1 + N_2 = N$. For group 1, we will denote the weakest channel gain as $G_{w1} \triangleq |H_1|^2 = \min\{|h_1|^2, |h_2|^2, ..., |h_{N_1}|^2\}$. In order to get analytically tractable results, we assume that all the sources are at normalized distance of $d = 1$ from the destination and hence all the source-destination channel gains are i.i.d with exponential distribution. The pdf of G_{w1} is given by $f_{G_{w1}}(g_{w1}) = N_1 e^{-N_1 g_{w1}}$ [30].

For group 1, we will denote the second weakest channel gain as G_{w2}. Since the channel gains are ordered according to their strengths, hence G_{w1} and G_{w2} are correlated. The joint pdf of G_{w2} and G_{w1} is given by [30]

$$f_{G_{w2},G_{w1}}(g_{w2}, g_{w1}) = N_1(N_1 - 1)e^{-(N_1-1)g_{w2}-g_{w1}} \tag{2.104}$$

For sake of clarity, for group 2, we will denote the weakest channel gain as $A_{w1} \triangleq |H_2|^2$

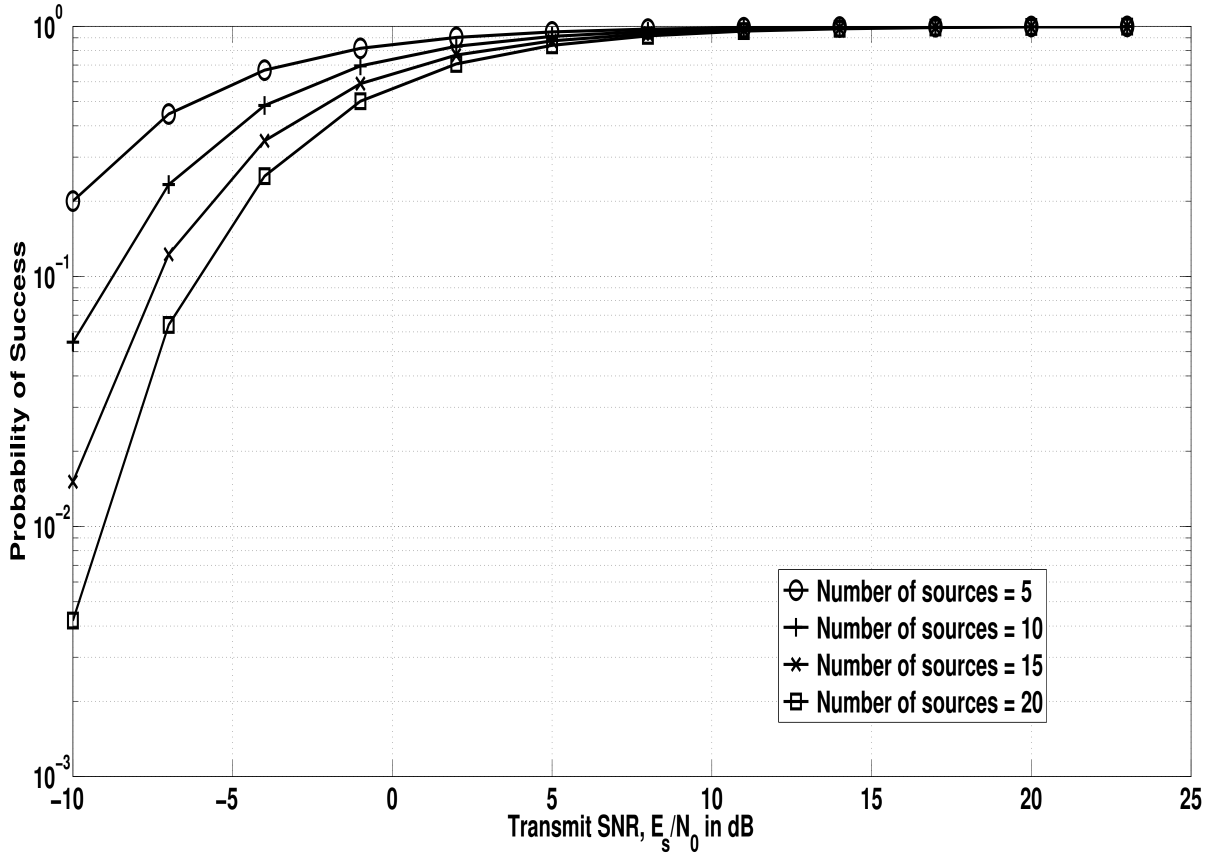

Figure 2.4 Probability of all sources being successfully decoded at the relay.

$= \min\{|h_{N_1+1}|^2, |h_{N_1+2}|^2, ..., |h_N|^2\}$. The pdf of A_{w1} is given by $f_{A_{w1}}(a_{w1}) = N_2 e^{-N_2 a_{w1}}$. For group 2, we will denote the second weakest channel gain as A_{w2}. The joint pdf of A_{w2} and A_{w1} is given by

$$f_{A_{w2},A_{w1}}(a_{w2}, a_{w1}) = N_2(N_2-1)e^{-(N_2-1)a_{w2}-a_{w1}} \tag{2.105}$$

The pdf of channel gain between relay and destination is given by $g_{r_1} \triangleq |h_{r_1}|^2 = \lambda_{r_1} e^{-\lambda_{r_1} g_{r_1}}$ where $\lambda_{r_1} = d_{r_1}^{\alpha}$.

The probability of occurrence of the event when all the sources are successfully decoded at the relay is

$$P(\mathcal{E}) = \prod_{i=1}^{N} P_{o,s_i r_1}^c = e^{-\lambda_{sr_1}\Gamma} \tag{2.106}$$

where

$$\Gamma = \frac{2^{(N+1)R/N} - 1}{E_s/N_0} \tag{2.107}$$

and $\lambda_{sr_1} = \sum_{i=1}^{N} d_{ir_1}^{\alpha}$. The factor of $(N+1)/N$ is to account for the fact that $N+1$ channel uses are performed in transmitting N packets. $P_{o,s_i r_1}$ is the outage probability of i^{th} source at the relay R_1 and $P_{o,s_i r_1}^c$ is the probability that i^{th} source is *not* in outage at the relay. We will derive the outage probability of both the groups individually.

Let $\mathcal{G}$ denotes the event $A_{w1} > \mu \cdot \min\{g_{r_1}, G_{w1}\}$ and $\mathcal{G}^c$ denotes the event $A_{w1} < \mu \cdot \min\{g_{r_1}, G_{w1}\}$. Since these two events are disjoint, therefore the outage probability of groups can be calculated for these individual events and then summed up to give total outage probability.

$$\begin{aligned} P_{o,1}(\mathcal{E}) &= P_{o,1}(\mathcal{G}) \cdot P(\mathcal{E}) + P_{o,1}(\mathcal{G}^c) \cdot P(\mathcal{E}) \\ P_{o,2}(\mathcal{E}) &= P_{o,2}(\mathcal{G}) \cdot P(\mathcal{E}) + P_{o,2}(\mathcal{G}^c) \cdot P(\mathcal{E}) \end{aligned} \tag{2.108}$$

2.4.8.1 Outage Analysis for Group 1

In this subsection, we will derive the outage probability for the weakest user of group 1. Since we derive the outage probability for weakest user, it serves as an upper bound to the outage probability for all the users of the group 1.

Under the event $\mathcal{G}$, $\mathbf{m}_{r_1} = \mathbf{M}_1$ is transmitted by the relay. The destination decodes message packets of group 1 using direct source-destination transmissions - $\mathbf{y}_1$, $\mathbf{y}_2$, ...,$\mathbf{y}_{N_1}$ and coded packet from relay, $\mathbf{z}_1$. Then the destination will not be able to decode at least the weakest user of group 1 if any of the following events occur:

1. The weakest user of Group 1 is in outage and the relay is in outage at destination. Let this event be denoted as $\mathcal{E}_{11}$.The probability of occurrence of this event is given by

$$P_{o,1}(\mathcal{E}_{11}) = Pr(\{G_{w1} < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G}) \tag{2.109}$$

2. Atleast 2 users of Group 1 are in outage at the destination and the relay is not in outage at the destination. The diversity order for three or more users begin in outage simultaneously at the destination and relay not in outage at the destination is atleast 3. Since the system under consideration has maximum diversity order of two, for calculation of outage probability we will consider the case when exactly two weakest users of Group 1 are in outage at the destination and the relay is not in outage. Let this event be denoted as $\mathcal{E}_{12}$.The probability of occurrence of this event is given by

$$P_{o,1}(\mathcal{E}_{12}) = Pr(\{G_{w1} < \Gamma\} \cap \{G_{w2} < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}) \tag{2.110}$$

Under the event $\mathcal{G}^c$, $\mathbf{m}_{r_1} = \mathbf{M}_1 \oplus \mathbf{M}_2$ is transmitted by the relay. The destination decodes the group 1 packets jointly using direct source-destination transmissions of group 1 and group 2 users and the network coded packet from the relay. The destination will not be able to decode atleast the weakest user from group 1 if any of the following events occur:

1. The weakest user of Group 1 is in outage and the relay is in outage at destination. Let this event be denoted as $\mathcal{E}_{13}$.The probability of occurrence of this event is given by

$$P_{o,1}(\mathcal{E}_{13}) = Pr(\{G_{w1} < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G}^c) \tag{2.111}$$

2. Exactly two weakest users of Group 1 are in outage at the destination and the relay is not in outage. Let this event be denoted as $\mathcal{E}_{14}$.The probability of occurrence of this event is given by

$$P_{o,1}(\mathcal{E}_{14}) = Pr(\{G_{w1} < \Gamma\} \cap \{G_{w2} < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}^c) \tag{2.112}$$

3. The weakest user of Group 1 is in outage, the weakest user of Group 2 is in outage, the second weakest user of Group 1 is not in outage and the relay is not in outage at the destination. Let this event be denoted as $\mathcal{E}_{15}$.The probability of occurrence of this event is given by

$$P_{o,1}(\mathcal{E}_{15}) = Pr(\{G_{w1} < \Gamma\} \cap \{A_{w1} < \Gamma\} \cap \{G_{w2} > \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}^c) \tag{2.113}$$

The total outage probability for the weakest user of Group 1 for event $\mathcal{E}$ can be derived by summing up the probabilities $P_{o,1}(\mathcal{E}_{11})$, $P_{o,1}(\mathcal{E}_{12})$, $P_{o,1}(\mathcal{E}_{13})$,$P_{o,1}(\mathcal{E}_{14})$,$P_{o,1}(\mathcal{E}_{15})$.

$$\begin{aligned} P_{o,1}(\mathcal{E}) = & \left(Pr(\{G_{w1} < \Gamma\} \cap \{g_{r_1} < \Gamma\}) + Pr(\{G_{w1} < \Gamma\} \cap \{G_{w2} < \Gamma\} \cap \{g_{r_1} > \Gamma\})\right. \\ & \left. + Pr(\{G_{w1} < \Gamma\} \cap \{A_{w1} < \Gamma\} \cap \{G_{w2} > \Gamma\} \cap \{A_{w1} < \mu G_{w1}\}) Pr(\{g_{r_1} > \Gamma\})\right) P(\mathcal{E}) \end{aligned} \tag{2.114}$$

After integration over appropriate limits, the outage probability for the weakest user of Group 1 in closed form can be derived as

$$\begin{aligned} P_{o,1}(\mathcal{E}) = & \left(\left(1 - e^{-N_1\Gamma}\right)\left(1 - e^{-\lambda_{r_1}\Gamma}\right) + e^{-\Gamma\lambda_r}\left(1 - e^{-\Gamma N_1} + N_1\left(e^{-\Gamma N_1} - e^{-\Gamma(N_1-1)}\right)\right)\right. \\ & \left. + \frac{N_1}{1+\mu N_2} e^{-\Gamma(N_1+\mu N_2+\lambda_r)}\left(1 - e^{\Gamma\mu N_2} - \mu N_2 e^{\Gamma\mu N_2} + \mu N_2 e^{(\Gamma\mu N_2+\Gamma)}\right)\right) e^{-\lambda_{sr_1}\Gamma} \end{aligned} \tag{2.115}$$

At high SNR, $E_s/N_0 \gg 1$, the second order approximation $e^{-x} \approx 1 - x + x^2/2$ yields the following asymptotic approximation for the outage probability for the weakest user of group 1 (assuming all the sources are decoded successfully at the relay)

$$P_{o,1} \sim \frac{1}{2}\Gamma^2\left(-N_1 + N_1^2 + \mu N_1 N_2 + 2N_1\lambda_r\right) \tag{2.116}$$

2.4.8.2 Outage Analysis for Group 2

In this subsection, we will derive the outage probability for the weakest user of group 2. Since we derive the outage probability for weakest user, it serves as an upper bound to the outage probability for all the users of the group 2.

Under the event $\mathcal{G}$, $\mathbf{m}_{r_1} = \mathbf{M}_1$ is transmitted by the relay. The destination decodes message packets of group 2 using direct source-destination transmissions - $\mathbf{y}_{N_1+1}$, $\mathbf{y}_{N_1+2}$, ..., $\mathbf{y}_N$. Then the destination will not be able to decode at least the weakest user of group 2 under the following event:

1. The weakest user of Group 2 is in outage at the destination. Let this event be denoted as $\mathcal{E}_{21}$.The probability of occurrence of this event is given by

$$P_{o,2}(\mathcal{E}_{21}) = Pr(\{A_{w1} < \Gamma\} \cap \mathcal{G}) \tag{2.117}$$

Under the event $\mathcal{G}^c$, $\mathbf{m}_{r_1} = \mathbf{M}_1 \oplus \mathbf{M}_2$ is transmitted by the relay. The destination decodes the group 2 packets jointly using direct source-destination transmissions of group 1 and group 2 users and the network coded packet from the relay. The destination will not be able to decode atleast the weakest user from group 2 if any of the following events occur:

1. The weakest user of Group 2 is in outage and the relay is in outage at destination. Let this event be denoted as $\mathcal{E}_{22}$.The probability of occurrence of this event is given by

$$P_{o,2}(\mathcal{E}_{22}) = Pr(\{A_{w1} < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{G}^c) \tag{2.118}$$

2. Exactly two weakest users of Group 2 are in outage at the destination and the relay is not in outage. Let this event be denoted as $\mathcal{E}_{23}$.The probability of occurrence of this event is given by

$$P_{o,2}(\mathcal{E}_{23}) = Pr(\{A_{w1} < \Gamma\} \cap \{A_{w2} < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}^c) \tag{2.119}$$

3. The weakest user of Group 2 is in outage, the weakest user of Group 1 is in outage, the second weakest user of Group 2 is not in outage and the relay is not in outage at the destination. Let this event be denoted as $\mathcal{E}_{24}$.The probability of occurrence of this event is given by

$$P_{o,2}(\mathcal{E}_{24}) = Pr(\{A_{w1} < \Gamma\} \cap \{G_{w1} < \Gamma\} \cap \{A_{w2} > \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{G}^c) \tag{2.120}$$

The total outage probability for the weakest user of Group 2 for event $\mathcal{E}$ can be derived by summing up the probabilities $P_{o,2}(\mathcal{E}_{21})$, $P_{o,2}(\mathcal{E}_{22})$, $P_{o,2}(\mathcal{E}_{23})$,$P_{o,2}(\mathcal{E}_{24})$. The resultant expression is too long to be presented here. We will perform numerical integration to calculate the exact outage probability for weakest user of Group 2.

At high SNR, $E_s/N_0 \gg 1$, the second order approximation $e^{-x} \approx 1 - x + x^2/2$ yields the following asymptotic approximation for the outage probability for the weakest user of group 2 (assuming all the sources are decoded successfully at the relay)

$$P_{o,2} \sim \Gamma^2 \frac{\left(\mu^2 N_1 N_2 + \mu N_2^2 - \mu N_2 + N_1 N_2 + \mu^2 N_2 \lambda_r + N_2 \lambda_r\right)}{2\mu} \tag{2.121}$$

Clearly our proposed scheme achieves maximal diversity order of 2 for both prioritized and non-prioritized group.

2.4.9 Outage Analysis for two source two relay

In this subsection, we will discuss about the outage probability of SPNC scheme for two-source multirelay scenario. We assume that there are two relays R_1 and R_2 in the system. Suppose S_1 is the prioritized source and S_2 is the non prioritized user.

For 2 sources and 2 relays, there will be 2^4 disjoint events to be analyzed for outage probability. The exact outage analysis for all these events is prohibitively complex and yields intractable results. In order to derive analytically tractable results, we will assume that both the sources are decoded successfully at all the relays. For mid to high SNR range, this event dominates the outage probability which we will denote by $\mathcal{E}$. The probability of occurrence of the event when all the sources are successfully decoded at both the relay is

$$P(\mathcal{E}) = \prod_{j=1}^{2} \prod_{i=1}^{2} P^c_{o,s_i r_j} \tag{2.122}$$

We have following disjoint cases to analyze

- *Case 1*: Let $\mathcal{C}_1$ denotes the event for which $|h_2|^2 > \mu \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 > \mu$ $|h_{r_2}|^2$. In this case, the relay R_1 transmits $\mathbf{m}_1$ and the relay R_2 transmits $\mathbf{m}_1$. The destination decodes message packets of source 1 using maximum likelihood decoding of direct source-destination transmissions - $\mathbf{y}_1$ and packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$. The destination will not be able to decode S_1 if $g_1 + g_{r_1} + g_{r_2} < \Gamma$, where $g_1 \triangleq |h_1|^2$, $g_{r_1} \triangleq |h_{r_1}|^2$ and $g_{r_2} \triangleq |h_{r_2}|^2$ and

$$\Gamma = \frac{2^{4R/2} - 1}{E_s/N_0} \tag{2.123}$$

The factor for 4/2 is because it takes 4 channel uses to transmit information for 2 users. Therefore the outage probability of source 1 under the event $\mathcal{C}_1$ is given by

$$\begin{aligned}P_{o,1}(\mathcal{C}_1)&=Pr(\{G_1+g_{r_1}+g_{r_2}<\Gamma\}\cap\mathcal{C}_1)\\&\le Pr(\{g_1<\Gamma\}\cap\{g_{r_1}<\Gamma\}\cap\{g_{r_2}<\Gamma\}\cap\mathcal{C}_1)\end{aligned} \tag{2.124}$$

The message of source 2 is decoded using signal received directly from the source, $\mathbf{y}_2$. Hence the outage probability of source 2 under the event $\mathcal{C}_1$ is given by

$$P_{o,2}(\mathcal{C}_1)=Pr(\{g_2<\Gamma\}\cap\mathcal{C}_1) \tag{2.125}$$

where $g_2 \triangleq |h_2|^2$.

- *Case 2*: Let $\mathcal{C}_2$ denotes the event for which $|h_2|^2 > \mu \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 < \mu$ $|h_{r_2}|^2$. In this case, the relay R_1 transmits $\mathbf{m}_1$ and the relay R_2 transmits $\mathbf{m}_1 \oplus \beta_2$ $\mathbf{m}_2$. Since R_2 transmits network coded packet including source 1 and source 2, hence the destination decodes message packets of source 1 using joint decoding of packets received via direct source-destination transmissions of source 1 - $\mathbf{y}_1$, packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$ and message packet of source 2, $\mathbf{y}_2$. Therefore the outage probability of source 1 under the event $\mathcal{C}_2$ is given by

$$\begin{aligned}P_{o,1}(\mathcal{C}_2)&=Pr(\{g_1+g_{r_1}<\Gamma\}\cap\{g_{r_2}<\Gamma\}\cap\mathcal{C}_2)\\&\quad+Pr(\{g_1+g_{r_1}<\Gamma\}\cap\{g_2<\Gamma\}\cap\{g_{r_2}>\Gamma\}\cap\mathcal{C}_2)\\&\le Pr(\{g_1<\Gamma\}\cap\{g_{r_1}<\Gamma\}\cap\{g_{r_2}<\Gamma\}\cap\mathcal{C}_2)\\&\quad+Pr(\{g_1<\Gamma\}\cap\{g_{r_1}<\Gamma\}\cap\{g_2<\Gamma\}\cap\{g_{r_2}>\Gamma\}\cap\mathcal{C}_2)\end{aligned} \tag{2.126}$$

Similarly, the destination decodes message packets of source 2 jointly using packet of direct source-destination transmissions - $\mathbf{y}_2$, packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$ and packets of source 1, $\mathbf{y}_1$. Therefore the outage probability of source 2 under the event $\mathcal{C}_2$ is given by

$$\begin{aligned}P_{o,2}(\mathcal{C}_2)&=Pr(\{g_2<\Gamma\}\cap\{g_{r_2}<\Gamma\}\cap\mathcal{C}_2)\\&\quad+Pr(\{g_2<\Gamma\}\cap\{g_1+g_{r_1}<\Gamma\}\cap\{g_{r_2}>\Gamma\}\cap\mathcal{C}_2)\\&\le Pr(\{g_2<\Gamma\}\cap\{g_{r_2}<\Gamma\}\cap\mathcal{C}_2)\\&\quad+Pr(\{g_2<\Gamma\}\cap\{g_1<\Gamma\}\cap\{g_{r_1}<\Gamma\}\cap\{g_{r_2}>\Gamma\}\cap\mathcal{C}_2)\end{aligned} \tag{2.127}$$

- *Case 3*: Let $\mathcal{C}_3$ denotes the event for which $|h_2|^2 < \mu \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 > \mu$ $|h_{r_2}|^2$. In this case, the relay R_1 transmits $\mathbf{m}_1 \oplus \beta_1\ \mathbf{m}_2$ and the relay R_2 transmits $\mathbf{m}_1$. The destination decodes message packets of source 1 using joint decoding of packets of direct source-destination transmissions - $\mathbf{y}_1$, packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$ and message packets of source 2 - $\mathbf{y}_2$. Therefore, the outage probability of source 1 under the event $\mathcal{C}_3$ is given by

$$\begin{aligned} P_{o,1}(\mathcal{C}_3) =& Pr(\{g_1 + g_{r_2} < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{C}_3) \\ &+ Pr(\{g_1 + g_{r_2} < \Gamma\} \cap \{g_2 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{C}_3) \\ \le& Pr(\{g_1 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{C}_3) \\ &+ Pr(\{g_1 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_2 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{C}_3) \end{aligned} \tag{2.128}$$

The destination decodes message packets of source 2 jointly using packets of direct source-destination transmissions - $\mathbf{y}_2$, packets from the relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$ and packets of source 1 $\mathbf{y}_1$. Therefore, the outage probability of source 2 under the event $\mathcal{C}_3$ is given by

$$\begin{aligned} P_{o,2}(\mathcal{C}_3) =& Pr(\{g_2 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{C}_3) \\ &+ Pr(\{g_2 < \Gamma\} \cap \{g_1 + g_{r_2} < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{C}_3) \\ \le& Pr(\{g_2 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \mathcal{C}_3) \\ &+ Pr(\{g_2 < \Gamma\} \cap \{g_1 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \mathcal{C}_3) \end{aligned} \tag{2.129}$$

- *Case 4*: Let $\mathcal{C}_4$ denotes the event for which $|h_2|^2 < \mu \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 < \mu$ $|h_{r_2}|^2$. In this case, the relay R_1 transmits $\mathbf{m}_1 \oplus \beta_1\ \mathbf{m}_2$ and the relay R_2 transmits $\mathbf{m}_1$ $\oplus\ \beta_2\ \mathbf{m}_2$.

  The destination decodes message packets of source 1 using joint decoding of packets of direct source-destination transmissions - $\mathbf{y}_1$, packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$ and message packets of source 2 - $\mathbf{y}_2$. Therefore the outage probability of source 1 under the event $\mathcal{C}_4$ is given by

$$\begin{aligned} P_{o,1}(\mathcal{C}_4) =& Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \mathcal{C}_4) \\ &+ Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_2 < \Gamma\} \cap \{g_{r_2} > \Gamma\} \cap \mathcal{C}_4) \end{aligned}$$

$$+Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \{g_2 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \mathcal{C}_4) \qquad (2.130)$$

Similarly the destination decodes message packets of source 2 using joint decoding of packets of direct source-destination transmissions - $\mathbf{y}_2$, packets from the relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$ and message packets of source 1 - $\mathbf{y}_1$. Therefore the outage probability of source 2 under the event $\mathcal{C}_4$ is given by

$$\begin{aligned}P_{o,2}(\mathcal{C}_4) =& Pr(\{g_2 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \mathcal{C}_4) \\ &+Pr(\{g_2 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_1 < \Gamma\} \cap \{g_{r_2} > \Gamma\} \cap \mathcal{C}_4) \\ &+Pr(\{g_2 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \{g_1 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \mathcal{C}_4) \qquad (2.131)\end{aligned}$$

Clearly the above four cases are disjoint. Hence the total outage probability of group 1 and group 2 can be calculated by summing the outage probabilities of all the disjoint events. Using laws of probabilities, the outage probability of source 1 can be simplified as

$$\begin{aligned}P_{o,1} \approx & [Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_{r_2} < \Gamma\}) \\ &+Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_2 < \Gamma\} \cap \{g_{r_2} > \Gamma\} \cap \{g_2 < \mu g_{r_2}\}) \\ &+Pr(\{g_1 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \{g_2 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_2 < \mu g_1\})]\, P(\mathcal{E}) \quad (2.132)\end{aligned}$$

The total outage probability of source 2 can be simplified as

$$\begin{aligned}P_{o,2} \approx & [Pr(\{g_2 < \Gamma\} \cap \{g_2 > \mu \min(g_1, g_{r_1})\} \cap \{g_2 > \mu g_{r_2}\}) \\ &+Pr(\{g_2 < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_2 > \mu \min(g_1, g_{r_1})\} \cap \{g_2 < \mu g_{r_2}\}) \\ &+Pr(\{g_2 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_2 < \mu \min(G_1, g_{r_1})\}) \\ &+Pr(\{g_2 < \Gamma\} \cap \{g_1 < \Gamma\} \cap \{g_{r_1} > \Gamma\} \cap \{g_{r_2} < \Gamma\} \cap \{g_2 < \mu g_1\}) \\ &+Pr(\{g_2 < \Gamma\} \cap \{g_1 < \Gamma\} \cap \{g_{r_1} < \Gamma\} \cap \{g_{r_2} > \Gamma\} \cap \{g_2 < \mu g_{r_2}\})]\, P(\mathcal{E}) \quad (2.133)\end{aligned}$$

After integrating over appropriate volumes and simplification, the outage probabilities of the source 1 and 2 can be derived as

$$\begin{aligned}P_{o,1}(\mathcal{E}) \approx & \Big[\left(1 - e^{-\lambda_1 \Gamma}\right)\left(1 - e^{-\lambda_{r_1} \Gamma}\right)\left(1 - e^{-\lambda_{r_2} \Gamma}\right) \\ &+ e^{-\lambda_{r_1} \Gamma}\left(1 - e^{-\lambda_{r_2} \Gamma}\right)\left(1 - e^{-\lambda_1 \Gamma} - \frac{\lambda_1}{\lambda_1 + \mu\lambda_2}\left(1 - e^{-\Gamma(\lambda_1 + \mu\lambda_2)}\right)\right)\end{aligned}$$

$$+\left(1-\frac{\lambda_1}{\lambda_1-\lambda_{r_1}}e^{-\lambda_{r_1}\Gamma}+\frac{\lambda_{r_1}}{\lambda_1-\lambda_{r_1}}e^{-\lambda_1\Gamma}\right)\left(e^{-\lambda_{r_2}\Gamma}\right.$$
$$\left.\left.-\frac{\lambda_{r_2}}{\mu\lambda_2+\lambda_{r_2}}e^{-\Gamma(\mu\lambda_2+\lambda_{r_2})}-\frac{\mu\lambda_2}{\mu\lambda_2+\lambda_{r_2}}e^{-\Gamma\left(\lambda_2+\frac{\lambda_{r_2}}{\mu}\right)}\right)\right]P(\mathcal{E})\quad(2.134)$$

$$P_{o,2}(\mathcal{E})\approx\left[2-e^{-\Gamma\lambda_2}-e^{-\Gamma\lambda_{r_2}}+\frac{\mu\lambda_2\left(e^{-\frac{\Gamma(\lambda_1+\mu\lambda_2+\lambda_{r_1})}{\mu}}-1\right)}{\lambda_2\mu+\lambda_1+\lambda_{r_1}}-\frac{\mu\lambda_2\left(e^{-\frac{\Gamma(\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2})}{\mu}}-1\right)}{\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2}}\right.$$
$$+\frac{\lambda_2\mu\left(e^{-\Gamma\lambda_{r_2}}-1\right)}{\lambda_2\mu+\lambda_1+\lambda_{r_1}}-\frac{\lambda_2\mu\lambda_{r_2}\left(e^{-\Gamma\left(\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2}\right)}-1\right)}{\left(\lambda_2\mu+\lambda_1+\lambda_{r_1}\right)\left(\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2}\right)}$$
$$+\frac{\lambda_{r_2}\left(e^{-\Gamma\left(\lambda_2\mu+\lambda_{r_2}\right)}-1\right)}{\lambda_2\mu+\lambda_{r_2}}+\frac{\mu\lambda_2\left(e^{-\frac{\Gamma(\lambda_2\mu+\lambda_{r_2})}{\mu}}-1\right)}{\lambda_2\mu+\lambda_{r_2}}$$
$$+\mu\lambda_2\left(1-e^{-\Gamma\lambda_{r_2}}\right)\left(\frac{1-e^{-\Gamma\lambda_{r_1}}}{\lambda_1+\mu\lambda_2+\lambda_{r_1}}-\frac{\lambda_{r_1}\left(1-e^{\Gamma(-\lambda_1-\mu\lambda_2)}\right)e^{-\Gamma\lambda_{r_1}}}{\left(\lambda_1+\mu\lambda_2\right)\left(\lambda_1+\mu\lambda_2+\lambda_{r_1}\right)}\right)$$
$$+\left(\frac{\lambda_1\left(e^{-\Gamma(\lambda_2\mu+\lambda_1)}-1\right)}{\lambda_2\mu+\lambda_1}-e^{-\Gamma\lambda_1}+1\right)e^{-\Gamma\lambda_{r_1}}\left(1-e^{-\Gamma\lambda_{r_2}}\right)$$
$$+\left(1-e^{-\Gamma\lambda_1}\right)\left(1-e^{-\Gamma\lambda_{r_1}}\right)\left(-\frac{\left(\lambda_2\mu e^{-\frac{\Gamma(\lambda_2\mu+\lambda_{r_2})}{\mu}}+\lambda_{r_2}e^{-\Gamma\left(\lambda_2\mu+\lambda_{r_2}\right)}\right)}{\lambda_2\mu+\lambda_{r_2}}\right)$$
$$\left.+\left(1-e^{-\Gamma\lambda_1}\right)\left(1-e^{-\Gamma\lambda_{r_1}}\right)e^{-\Gamma\lambda_{r_2}}\right]P(\mathcal{E})\qquad(2.135)$$

where $\lambda_1 = d_1^\alpha$, $\lambda_2 = d_2^\alpha$ $\lambda_{r_1} = d_{r_1}^\alpha$ $\lambda_{r_2} = d_{r_2}^\alpha$. As shown by simulation results, the analysis matches closely with simulation results. At high SNR, $E_s/N_0 \gg 1$, the third order approximation $e^{-x} \approx 1 - x + x^2/2 - x^3/6$ yields the following asymptotic approximation for the outage probabilities:

$$P_{o,1} \approx \Gamma^3\left(\lambda_1\lambda_{r_1}\lambda_{r_2}+\frac{\mu\lambda_1\lambda_2\lambda_{r_2}}{2}+\frac{\lambda_1\lambda_2\lambda_{r_1}}{2}\right)\qquad(2.136)$$
$$P_{o,2} \approx \Gamma^3\left(\lambda_1\lambda_2\lambda_{r_1}+\frac{2}{3}\mu\lambda_1\lambda_2\lambda_{r_2}+\frac{2}{3}\mu\lambda_2\lambda_{r_1}\lambda_{r_2}+\frac{\lambda_1\lambda_2\lambda_{r_2}}{3\mu^2}+\frac{\lambda_2\lambda_{r_1}\lambda_{r_2}}{3\mu^2}\right)\qquad(2.137)$$

Clearly the proposed scheme achieves maximal diversity order of 3.

2.5 Optimizing μ and Relay Location

Typically the QoS requirements are specified by the maximum acceptable outage probability for a given receive SNR and rate. To optimize μ and the relay location, Eqs.(2.75) and (2.76) can be used to formulate a multi-objective optimization problem:

$$\begin{aligned} \min_{\mu,\delta,\beta} \quad & \tilde{P}_{O,1}, \quad \tilde{P}_{O,2} \\ \text{s.t} \quad & 0 < \tilde{P}_{O,1} \leq P^*_{O,1};\ 0 < \tilde{P}_{O,2} \leq P^*_{O,2} \\ & (\beta)^{1/\alpha} + (\delta)^{1/\alpha} > 1 \\ & 0 < \mu \leq 1;\ 0 < \beta \leq 1;\ 0 < \delta \leq 1 \end{aligned} \tag{2.138}$$

where $P^*_{O,1}$ and $P^*_{O,2}$ are the maximum outage probability constraints. The constraint $(\beta)^{1/\alpha}$ + $(\delta)^{1/\alpha} > 1$ is the triangle inequality that the sum of two sides of a triangle are greater than the third side and we have assumed that the sources are at normalized distance of 1 from the destination in Eqs.(2.75) and (2.76). The approximate outage probabilities given by Eqs.(2.75), (2.76) are smooth functions in $\mu > 0, \delta > 0, \beta > 0$. Hence the above optimization problem can be solved easily using standard constrained optimization tools [32].

Sometimes, the optimization may not yield any valid solution. Then the optimization problem can be reformulated to minimize $\tilde{P}_{O,2}$ under the constraint $0 < \tilde{P}_{O,1} \leq P^*_{O,1}$. This way we can satisfy QoS constraint for prioritized user and simultaneously provide best possible QoS to the other user. In such cases we can reformulate the above optimization problem as following:

$$\begin{aligned} \min_{\mu,\delta,\beta} \quad & \tilde{P}_{O,2} \\ \text{s.t} \quad & 0 < \tilde{P}_{O,1} \leq P^*_{O,1}; 0 < \tilde{P}_{O,2} < 1 \\ & (\beta)^{1/\alpha} + (\delta)^{1/\alpha} > 1 \\ & 0 < \mu \leq 1,\ 0 < \beta \leq 1;\ 0 < \delta \leq 1 \end{aligned} \tag{2.139}$$

This formulation yields best possible throughput for non-prioritized source while satisfying the QoS constraint for the priority source. This way the joint optimization of δ and μ can be performed to satisfy QoS constraints where the optimization of δ tells the optimum location of the relay.

For optimizing μ and δ, we consider the QoS constraints of $P^*_{out,1} \leq 10^{-4}$ and $P^*_{out,2} \leq 10^{-3}$ at the transmission rate of 0.5 bits per channel use for each user and transmit SNR, $E_s/N_0 =$ 20 dB. By using (2.138) we found that these constraints can indeed be satisfied. The δ was found out to be 0.0902 implying $d_{r_1 d} = (0.0902)^{1/4} = 0.5481$, μ was found to be 0.2663 and $\beta =$

0.0455 implying $d_{sr_1} = 0.4619$. By using exact outage probability expressions (2.73) and (2.74) we can verify that these values do satisfy the given QoS constraints. Now suppose that the QoS constraints are changed to $P^*_{out,1} \leq 5 \times 10^{-6}$ and $P^*_{out,2} \leq 10^{-4}$ keeping the same transmit SNR and spectral efficiency. By using (2.138) we were not able to find a valid solution for μ, δ and β which satisfy the constraint. By changing the formulation to (2.139), we can find the optimum μ to be 0.0348, δ to be 0.0988, implying $d_{r_1 d} = (0.0988)^{1/4} = 0.5606$ and β to be 0.0408, implying $d_{sr_1} = 0.4494$, which gives $P_{out,1} = 4.9823 \times 10^{-6}$ and $P_{out,2} = 6.8353 \times 10^{-4}$ satisfying QoS constraint for prioritized user and best possible throughput for the other user.

2.6 Numerical Results

In this section we present numerical results. The path loss exponent is assumed to be $\alpha = 4$. The transmit energies of all the sources and relays are assumed to be the same.

First we assume two sources S_1 and S_2 both are at a distance of 1 from the destination D, and that the relay node R_1 is at a distance of 0.6 from D and approximately at a distance of 0.5 from the sources. We assume that S_1 has a higher priority than S_2. Fig.2.5 shows the outage performance of SPNC and perfect channel estimation versus transmit SNR, E_s/N_0. The diversity order for both S_1 and S_2 is two. We can see that the theoretical analysis of Eqs.(2.73) and (2.74), and the approximate asymptotic analysis of Eqs.(2.75) and (2.76) matches quite closely with simulation results. We assume spectral efficiency of $R = 0.5$ bits per channel use.

Fig.2.6 shows the outage probability of SPNC and traditional NC versus rate for $\mu =$ 0.05 and 0.2 at transmit SNR E_s/N_0 of 20 dB. We can see that the outage probability of the prioritized source S_1 increases while that of the non-prioritized source S_2 decreases with increasing scale-factor μ. Hence, with different values of μ, one can achieve different levels of reliability. An interesting observation is that the rate gain for the prioritized node S_1 over traditional NC is more than the rate loss for non-prioritized node S_2 over traditional NC. This leads to an overall rate gain for the SPNC scheme compared to the traditional NC. This is because whenever the channel from S_2 to D is sufficiently strong, the relay forwards the data from S_1 only to the destination. This leads to a significant rate increase for S_1 especially for the case where the channel gain from S_1 to D is weak. At the same time, since the channel

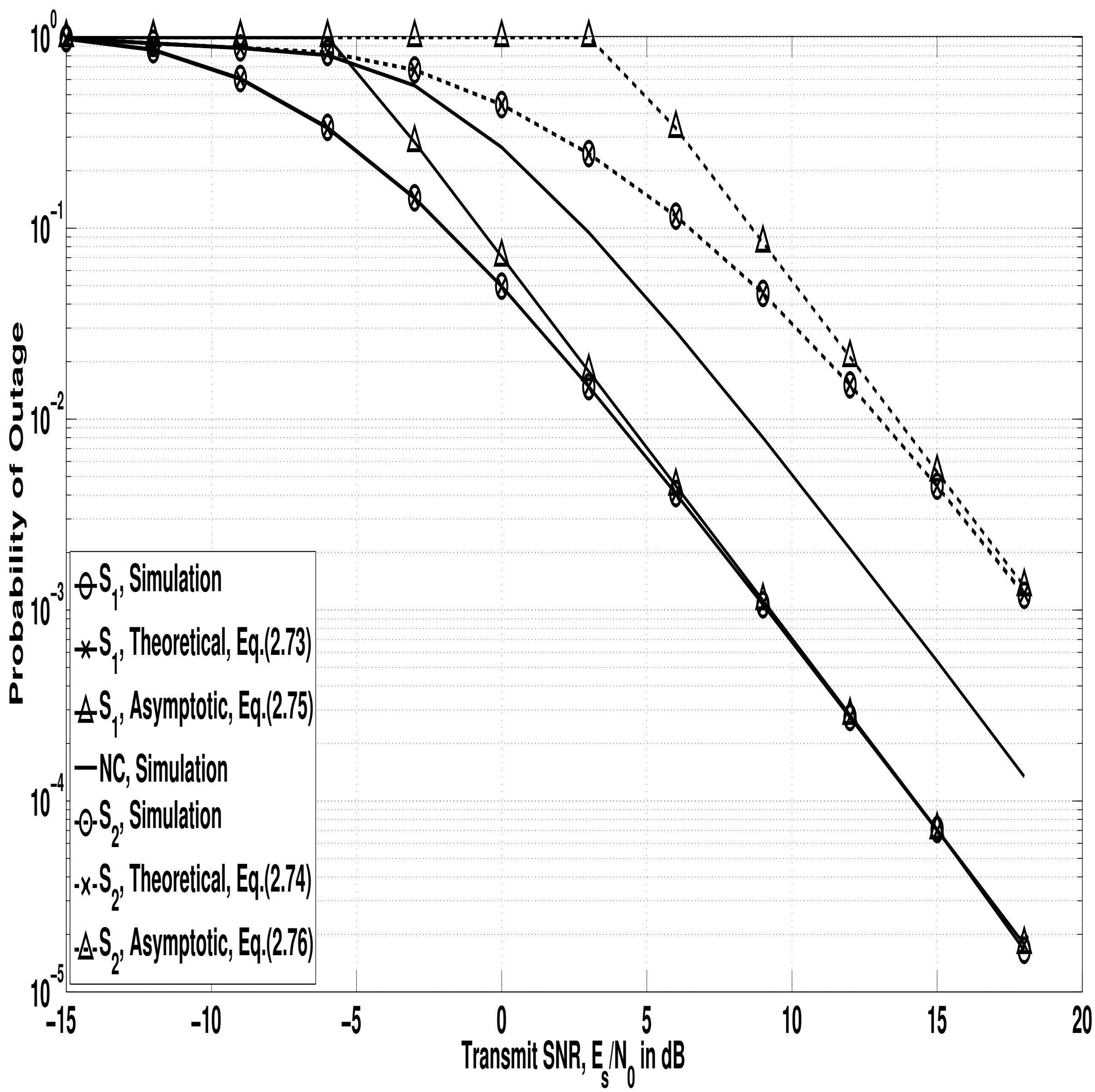

Figure 2.5 Probability of outage for SPNC for 2 sources for $\mu = 0.05$.

from S_2 to D is sufficiently strong, the loss in rate for S_2 by excluding it from the network coding is quite small. Hence, there is an overall rate gain by using SPNC. As shown by the figure, by using different μ, different QoS can be provided to the users.

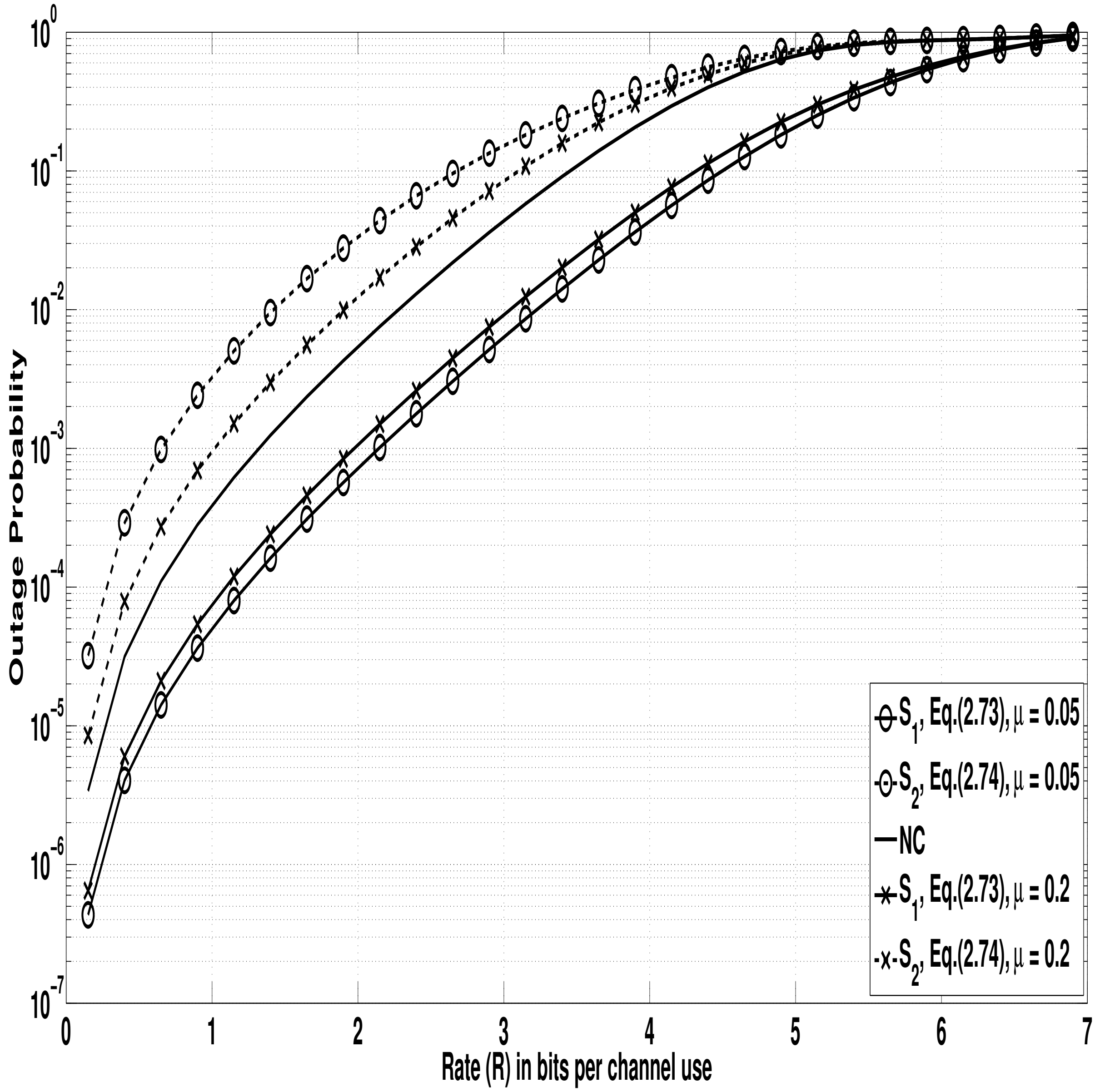

Figure 2.6 Probability of outage for SPNC versus Rate (R) at Transmit SNR, $E_s/N_0 = 20$dB.

Fig. 2.7 shows the bit error probability with SPNC against the transmit SNR, E_s/N_0 assuming perfect channel estimation. For simulation purpose, we have assumed that the sources are decoded correctly at the relay. We can see that the bit error probability of prioritized source

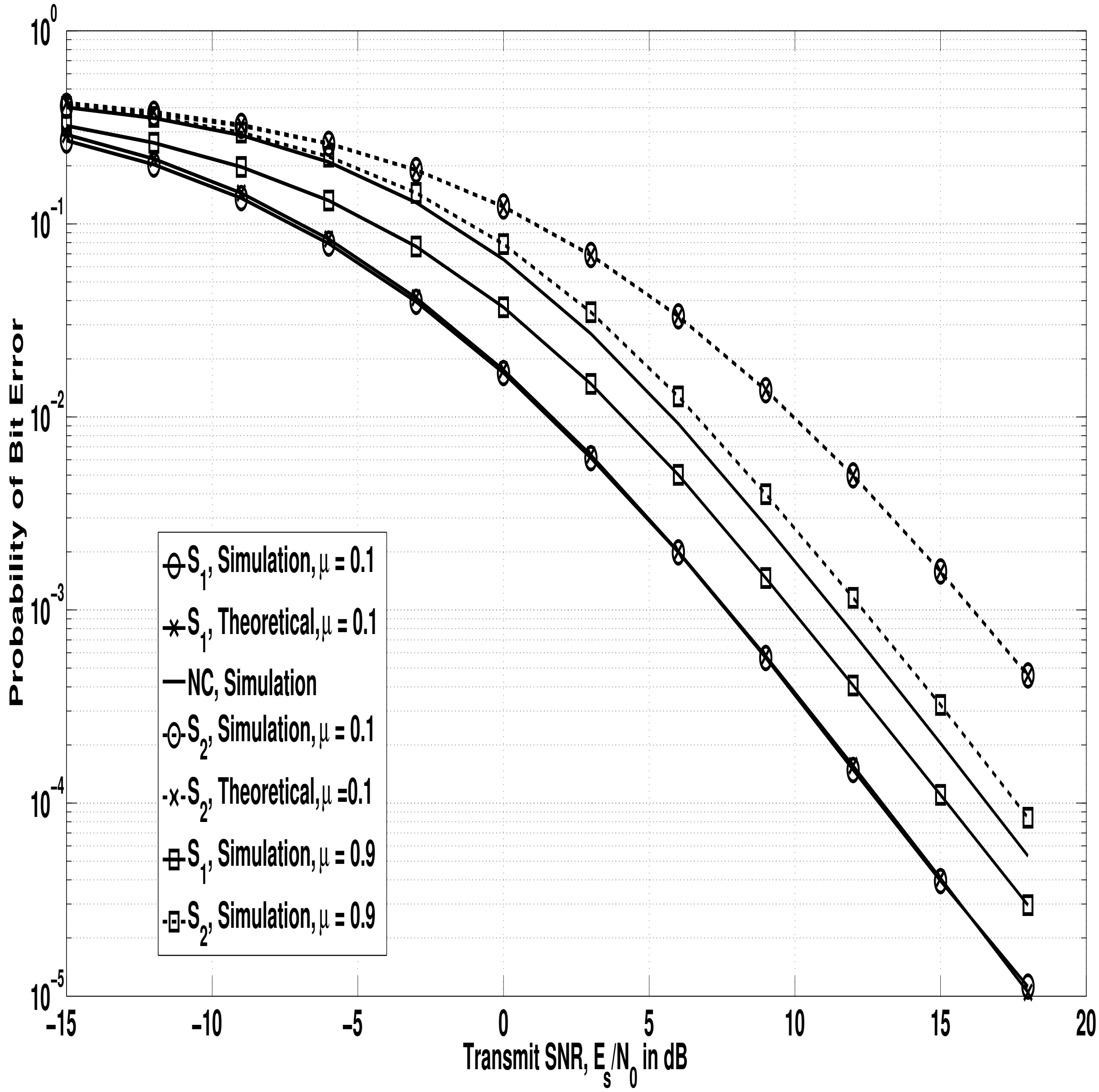

Figure 2.7 Probability of bit error with SPNC for 2 sources, $\mu = 0.1$, 0.9, Analytical for S_1 - Eqs. (2.17), (2.21), (2.22), (2.23), (2.24), (2.27), (2.28) and Analytical for S_2 - Eqs. (2.17), (2.27), (2.28), (2.31), (2.32), (2.33), (2.34)

S_1 increases while that of non-prioritized source S_2 decreases with increasing scale-factor μ. The diversity order for both S_1 and S_2 is 2. Hence, with different values of μ, one can achieve different levels of performance without a loss in diversity for either user. This leads to a smaller performance loss for S_2 for a given performance gain for S_1, which will result in a higher overall throughput. We can also see that the theoretical analysis matches quite closely with simulation results. The numerical integration was performed since exact closed form expression cannot be derived.

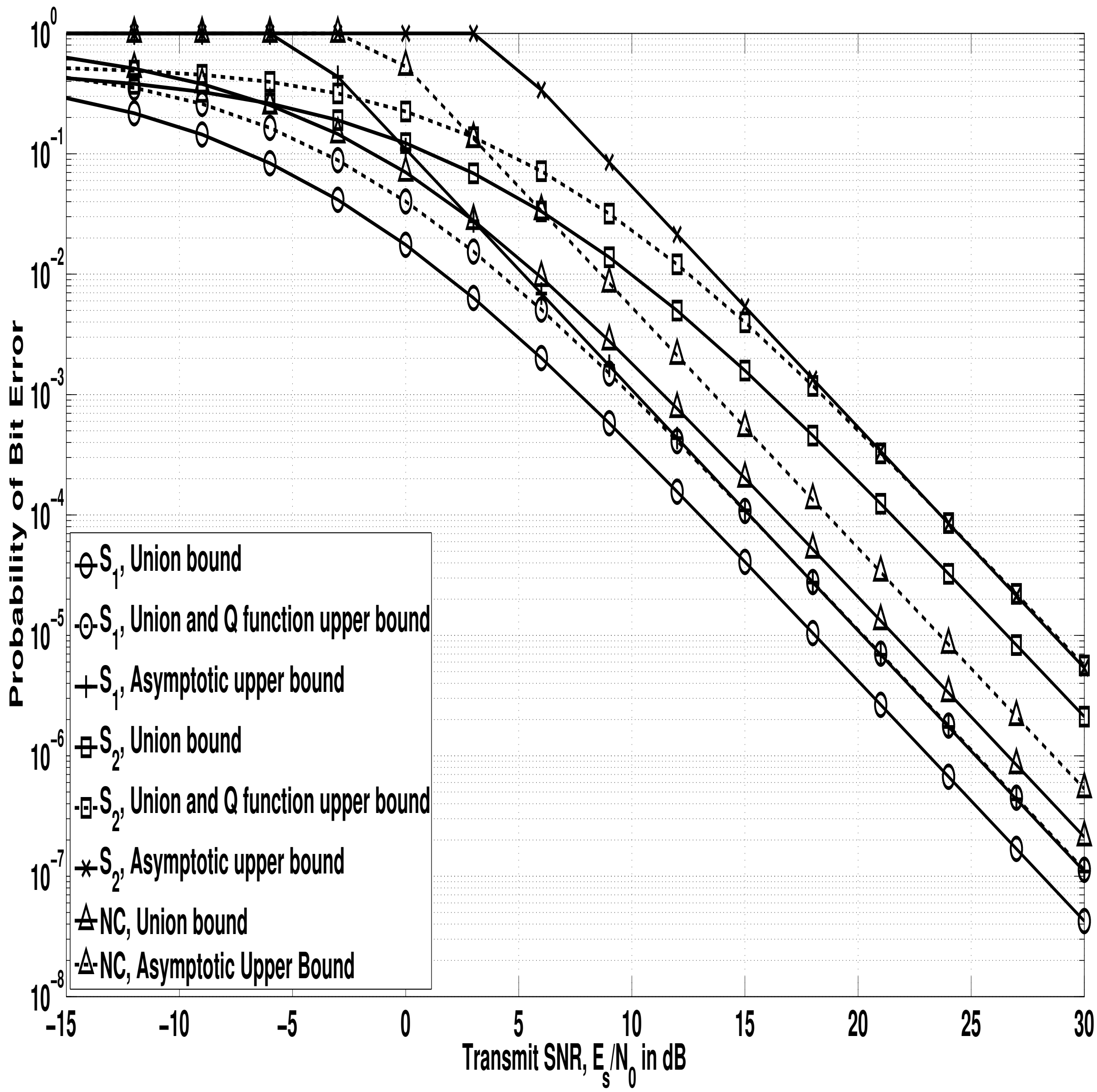

Figure 2.8 SPNC asymptotic bounds for $\mu = 0.1$

Fig. 2.8 shows the comparison of the bit error probability of S_1 and S_2 for SPNC scheme evaluated by using union and upper bound on Q function for $\mu = 0.1$. For S_1's union and Q function upper bound, we use Eqs.(2.17), (2.38), (2.39), (2.40), (2.41), (2.43), (2.44). For S_2's union and Q function upper bound, we use Eqs.(2.17), (2.43), (2.44), (2.47), (2.48), (2.49), (2.50). For network coding union and Q function upper bound, we use Eq. (2.58). For S_1's asymptotic bound on bit error probability we use equation Eq.(2.53). For S_2's asymptotic bound we use Eq.(2.56). For network coding's asymptotic bound, we use Eq.(2.59). The union bound for S_1 is analytical derivation for S_1 using Eqs. (2.17), (2.21), (2.22), (2.23), (2.24), (2.27), (2.28) and their numerical integration. The union bound for S_2 is the analytical derivation for S_2 using Eqs. (2.17), (2.27), (2.28), (2.31), (2.32), (2.33), (2.34) and their numerical integration. As shown in the figure, the Q function upper bounds and asymptotic bounds are tight and is within 2 dB of exact bit error probability.

Fig. 2.9 shows the probability of bit error for SPNC versus the transmit SNR, E_s/N_0, with channel estimation error. It does introduces small performance loss because of estimation error. However there is no loss in diversity order of the 2 sources and hence SPNC scheme can be used effectively for prioritization in practice. The analytical bound for bit error probability under channel estimation error for S_1 is derived by using Eq.2.63 which replaces γ_i in Eqs. (2.17), (2.21), (2.22), (2.23), (2.24), (2.27), (2.28) and then performing numerical integration. Similarly for S_2 is derived by using Eq.2.63 which replaces γ_i in Eqs. (2.17), (2.27), (2.28), (2.31), (2.32), (2.33), (2.34) and then performing numerical integration.

Now we consider three sources, one relay and one destination case. All the three sources are at normalized distance 1 and the relay is at a distance of 0.6 from the destination. The three sources, S_1, S_2 and S_3 are approx at a distance of 0.42, 0.4, 0.42 from the relay respectively. Fig. 2.10 shows the outage probability for 3 sources with different priorities, S_1 being at the highest priority and S_3 at the lowest. The SPNC rule used is Eq.(2.12). As shown by simulation, SPNC can be used effectively to provide different QoS to different users by choosing different values of μ_2 and μ_3. We assume spectral efficiency of $R = 0.5$ bits per channel use.

Now we consider 10 sources, one relay and one destination with cluster based grouping. We assume two groups, each containing 5 sources each. All the sources and group head are

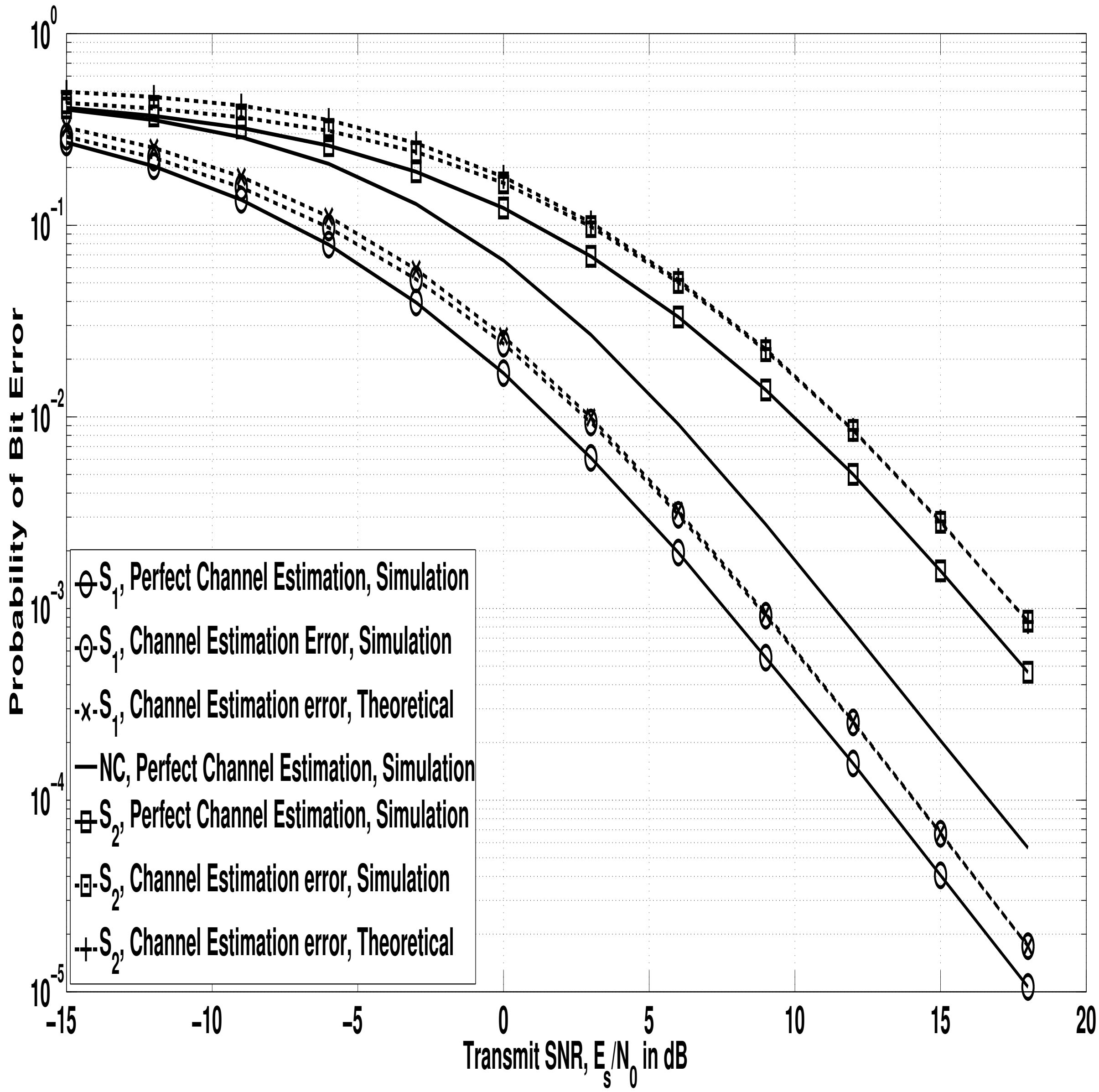

Figure 2.9 Probability of bit error for SPNC with channel estimation error, $\mu = 0.1$, L = 10, $\omega_{ND} = 0.04\pi$.

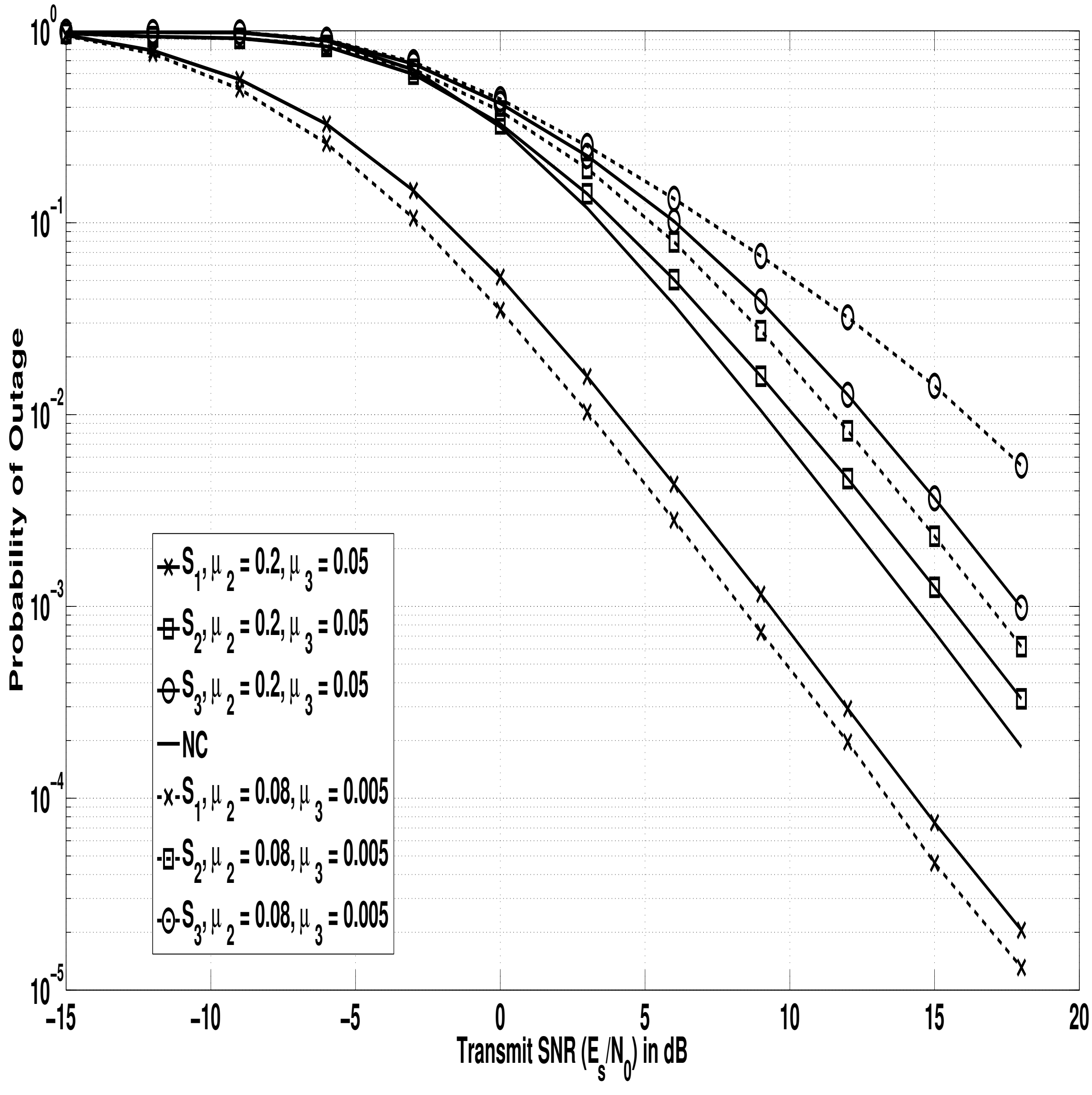

Figure 2.10 Probability of outage for SPNC with 3 sources, one relay.

at normalized distance 1 and the relay is at a distance of 0.6 from the destination. Both the groups are at equal distances 0.5 from the relay. As shown by the simulation results for Fig. 2.11, SPNC can be used to provide different QoS to different groups. For the case when group heads are equidistant from relay and destination, the analysis for 2 source, 1 relay can be used to derive the outage performance for each group with appropriate changes to Γ. We assume spectral efficiency of $R = 0.5$ bits per channel use.

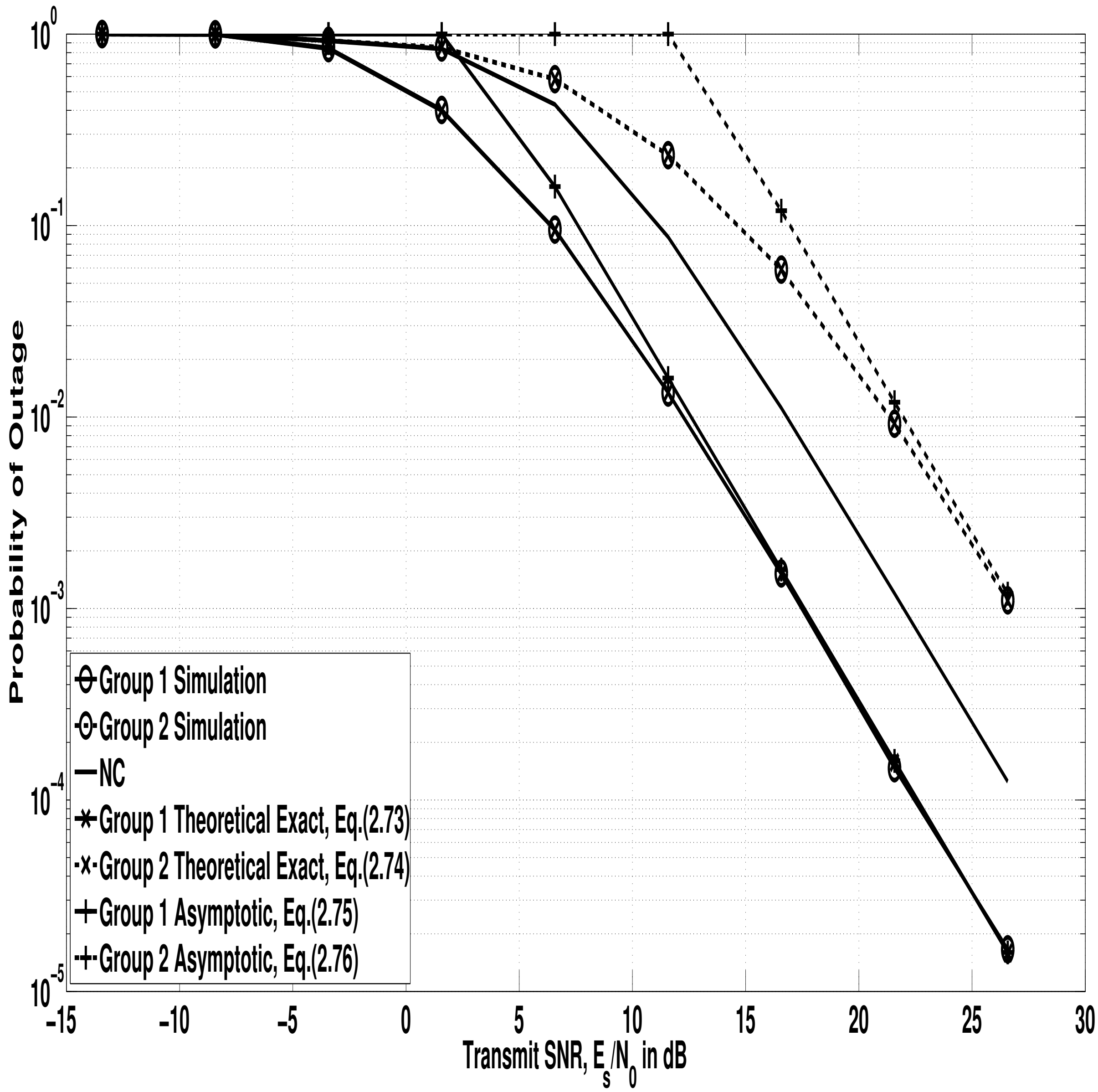

Figure 2.11 Probability of outage for SPNC with cluster based grouping and group heads at equal distances from relay and destination, $\mu = 0.05$.

We again consider 10 sources, one relay and one destination with cluster based grouping, however with unequal distances of group heads to the destination and relay. We assume two groups, each containing 5 sources each. We assume that the sources for group 1 and group 1 head are at normalized distance 0.9 from the destination and at distance of 0.4 from the relay. We assume that the sources for group 2 and group 2 head are at normalized distance 1 from the destination and at distance of 0.5 from the relay. As shown by the simulation results for Fig. 2.12, SPNC can be used to provide different QoS to different groups. The exact theoretical outage probability for group 1 is derived by numerical integration of Eqs. (2.83), (2.89) and summing them up with Eqs.(2.94), (2.96), (2.98). The upper bound for group 1 is given by Eq.(2.100). The exact outage probability for group 2 is calculated using Eq.(2.101). The asymptotic outage probability for group 1 is given by Eq.(2.102) and for group 2 is given by Eq.(2.103). We assume spectral efficiency of $R = 0.5$ bits per channel use.

Now we present results for normal grouping. We assume 7 users divided into 2 groups. The prioritized group, group 1 has 3 users and group 2 has 4 users. All users are at normalized distance of one from the destination and the relay is assumed at the normalized distance of 0.6 from destination. For simulation purpose, we assume that all users are decoded successfully at the relay. The exact theoretical outage probability for group 1 is given by Eq.(2.115). We perform numerical integration to determine the exact outage probability for non prioritized group. The asymptotic outage probability for group 1 is given by Eq.(2.116) and for group 2 is given by Eq.(2.121). As shown by the results in Fig. 2.13 and Fig. 2.14, with different μ, different reliability can be provided to different groups. We assume spectral efficiency of $R = 0.5$ bits per channel use.

We consider two sources, S_1 and S_2, two relays R_1 and R_2 and one destination. All the sources are at distance of 1 from destination and the relays R_1 and R_2 are at a distance 0.4 and 0.6 from the destination. The sources S_1 and S_2 are at a distance of 0.61 from R_1 and 0.42 from R_2 respectively. The sources are located such that they are equidistant from both the relays and destination. As shown by the simulation results in Fig. 2.15 and Fig. 2.16, with different μ, different reliability can be provided to different user. By using proposed SPNC, maximal diversity order, which is 3 in this case, can be provided to both the users. For simulation,

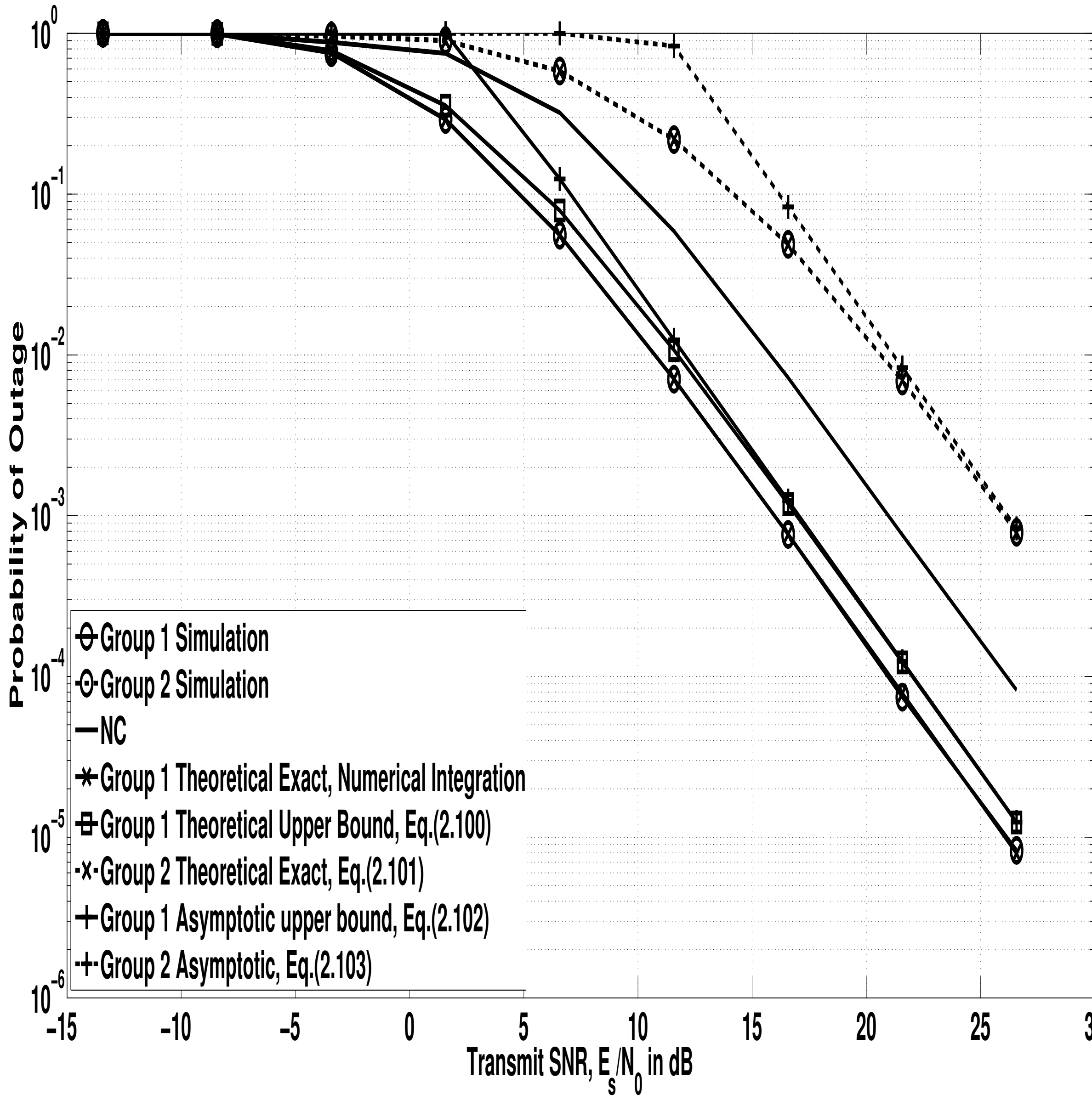

Figure 2.12 Probability of outage for SPNC with cluster based grouping and group heads at unequal distances from relay and destination, $\mu = 0.05$.

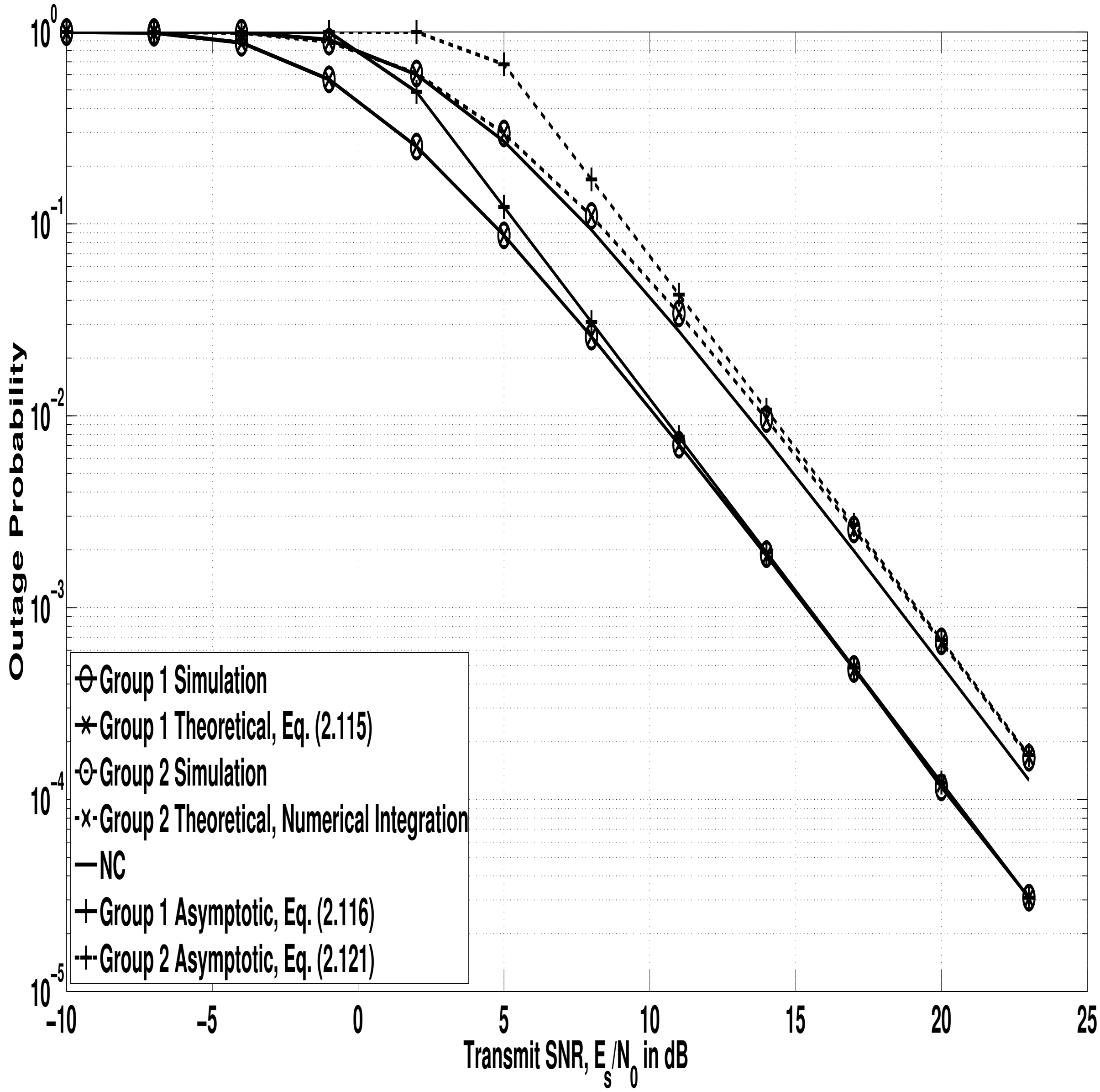

Figure 2.13 Probability of outage for SPNC with 2 groups, 1 relay and one destination, $\mu = 0.3$.

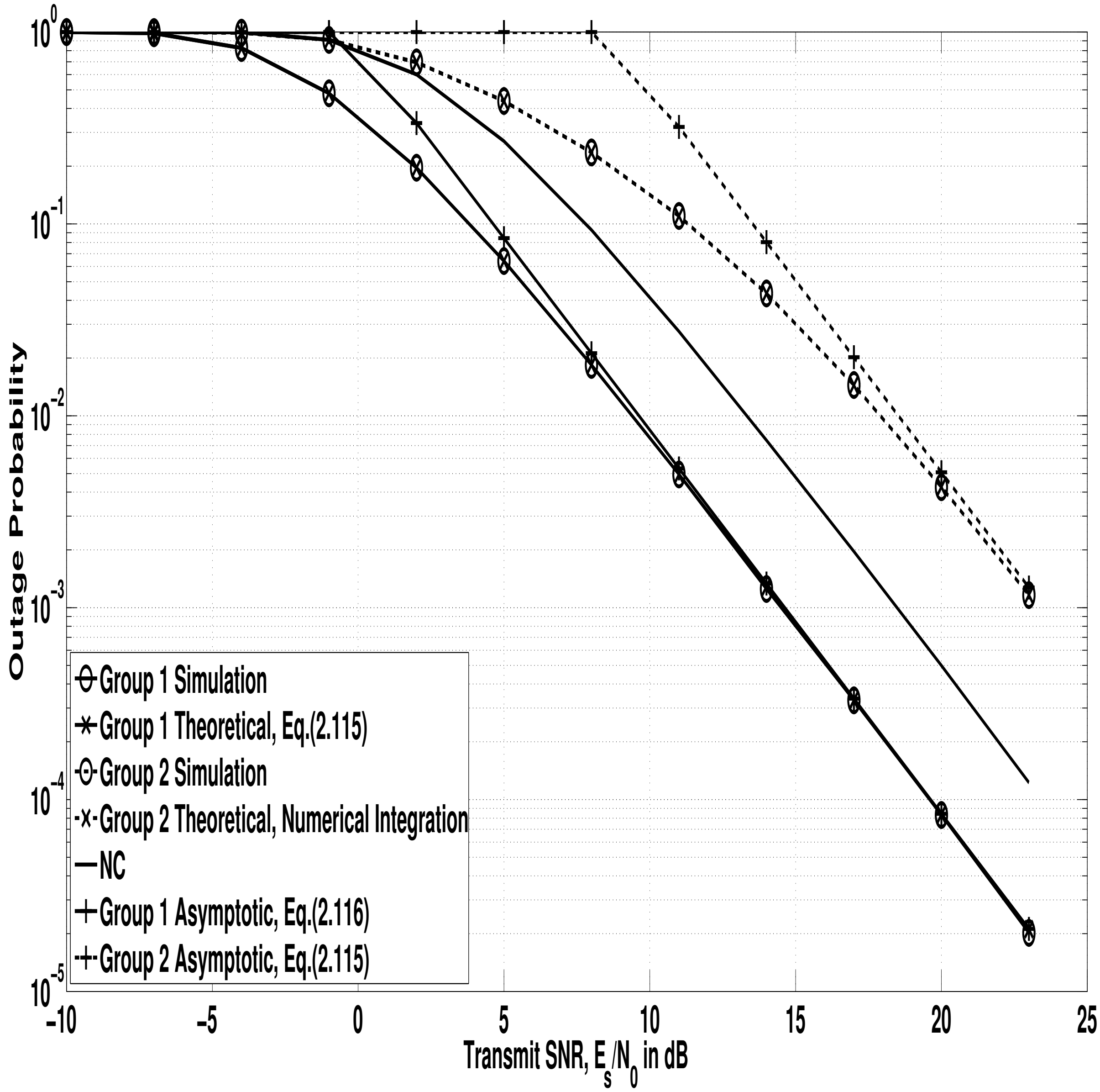

Figure 2.14 Probability of outage for SPNC with 2 groups, 1 relay and one destination, $\mu = 0.03$.

we assume that all the sources are decoded successfully at the relays. The simulation results match well the approximate theoretical bounds given by Eqs. (2.134), (2.135) and asymptotic bounds given by Eqs.(2.136) and (2.137). We assume spectral efficiency of $R = 0.5$ bits per channel use.

2.7 Conclusion

We proposed a soft prioritized network coding technique that enable a soft-level prioritized service to different nodes in wireless multiple access relay network. The proposed technique utilizes the channel state information of source-to-destination links in determining the network encoding rule at the relay. We showed that the proposed technique can provide soft-level prioritized services, in terms of rate and reliability, depending on the assistance needs. Given that we expect a growing need for variable user-specific service, the proposed techniques can provide a user tailored service that brings fine-tuned user satisfaction.

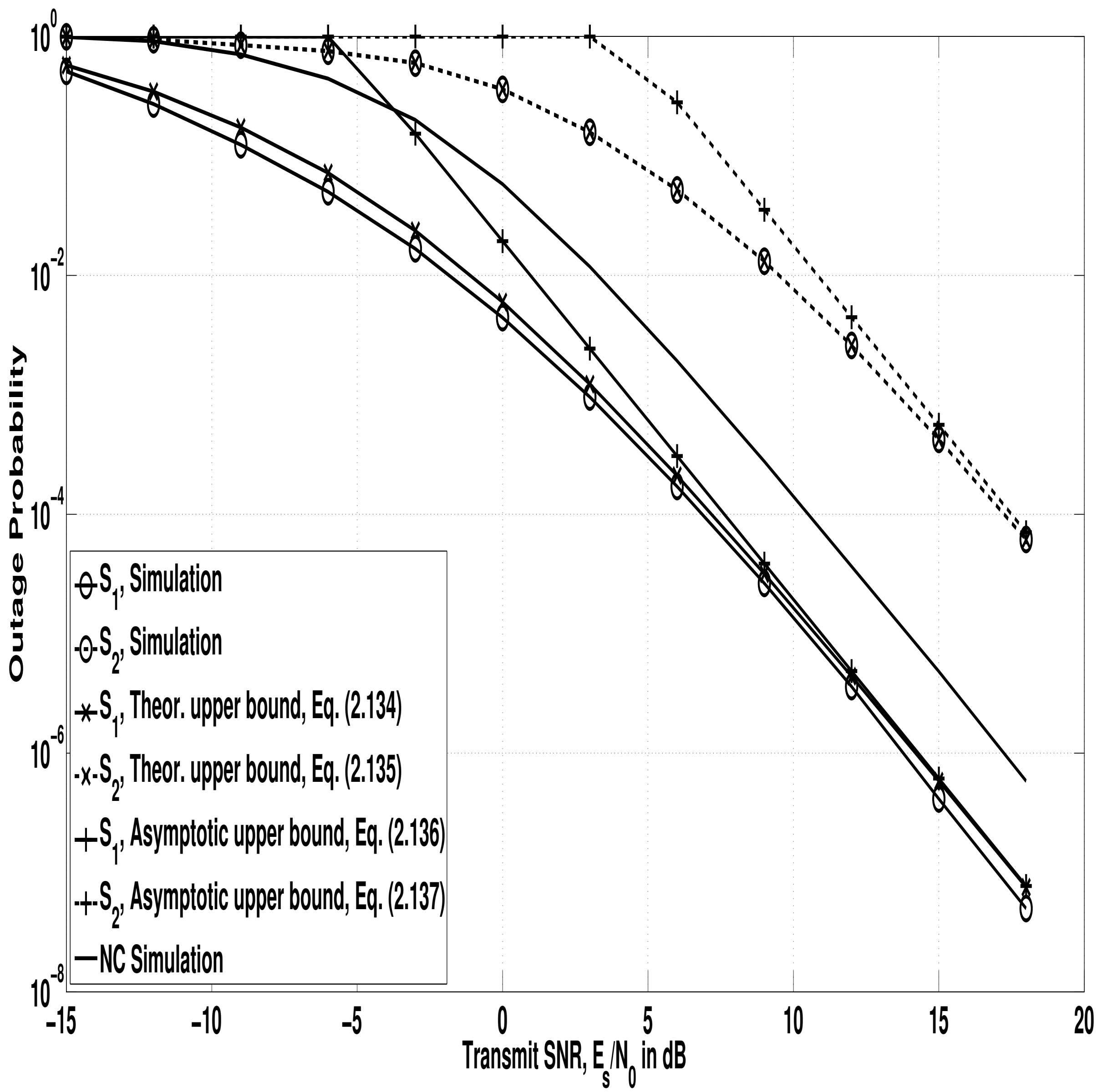

Figure 2.15 Probability of outage for SPNC with 2 users, 2 relays and one destination, $\mu = 0.05$. Theoretical for S_1- Eq.(2.134), Theoretical for S_2 - Eq.(2.135), Asymptotic for S_1 and S_2 - Eqs.(2.136) and (2.137).

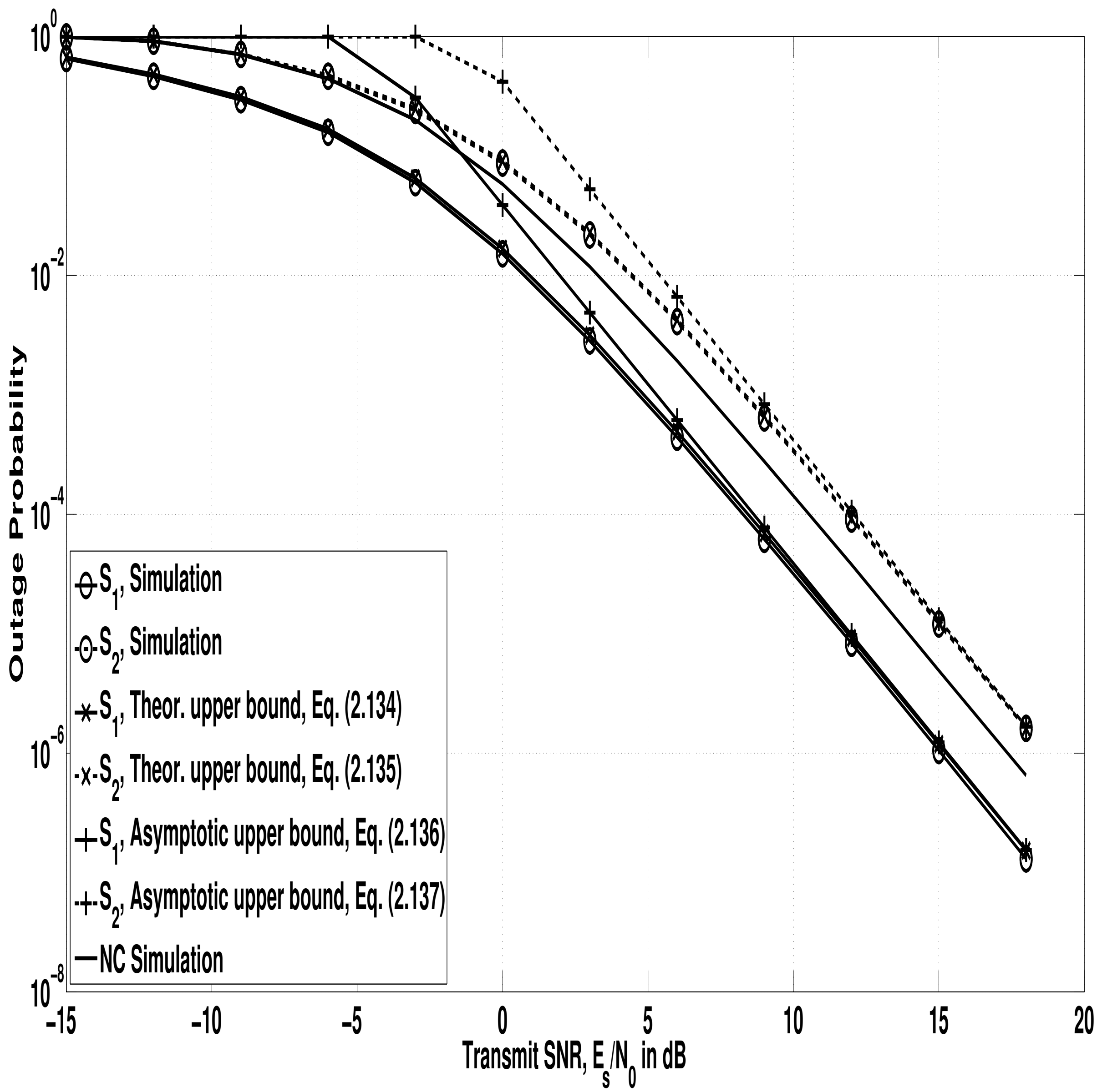

Figure 2.16 Probability of outage for SPNC with 2 users, 2 relays and one destination, $\mu = 0.35$. Theoretical for S_1- Eq.(2.134), Theoretical for S_2 - Eq.(2.135), Asymptotic for S_1 and S_2 - Eqs.(2.136) and (2.137).

CHAPTER 3. ITERATIVE RECYCLING OF POLLUTED PACKETS IN WIRELESS NETWORK CODING

3.1 Introduction

With the advent of large wireless networks, security has emerged as one of the prime concerns in their design. Wireless network coded systems are vulnerable to pollution attacks where a malicious node injects false information in its transmission to degrade the decoding capability. In case of multiple access relay network (MARN) with network coding (NC), the destination uses the network coded packets generated by relay nodes in addition to the message packets received directly from sources to improve the reliability of decoding [48; 49; 50]. An adversary can alter the data relayed by the wireless nodes and the performance of the decoding at the destination degrades drastically. In multihop case, the polluted packet may propagate to entire network. Cryptographic defenses to these problems are based on homomorphic signatures and message authentication codes (MAC) which can detect false injection and discard the polluted packet(s). The problem of detecting malicious relay nodes in single-source, multi-relay networks has been studied extensively in the literature for different relaying strategies [51; 52; 53; 54; 55; 56]. In [51] a signature based scheme is proposed to check the integrity of received packets. In [54; 55] several information theoretic algorithms for mitigating falsified data injection effects are proposed. In [52] the authors consider inserting a number of tracing bits in the data stream at the source in a cryptographically secure manner in single source scenario. The receiver then computes the ground truth of the tracing bits and compares them with the tracing bits received from the relay path to determine whether a relay node is adversarial or cooperative. If the correlation between them is above a threshold then we decide that the relay node is cooperative otherwise, it is malicious. The authors of [53] propose a statistical detection technique in order to mitigate malicious behavior in adaptive decode and forward (DF) cooperative diversity. The polluted packets are discarded and not used for further decoding in most of the earlier works.

In this work we consider *recycling* the polluted packet to restore the true coded packet and thereby improve the reliability of the decoded codeword at the destination. We propose an iterative packet recycling scheme that detects and removes the falsely injected packet in an iterative manner to enhance the reliability of decoding in the presence of pollution attack. This approach is motivated by the notion that the increased reliability after decoding can reduce the uncertainty about the falsely injected packet. This results in accurate detection and removal of false injection which can help increase the reliability of decoding. We show that the proposed scheme can provide a significant improvement over the traditional schemes that discard polluted packets prematurely.

3.2 System Model

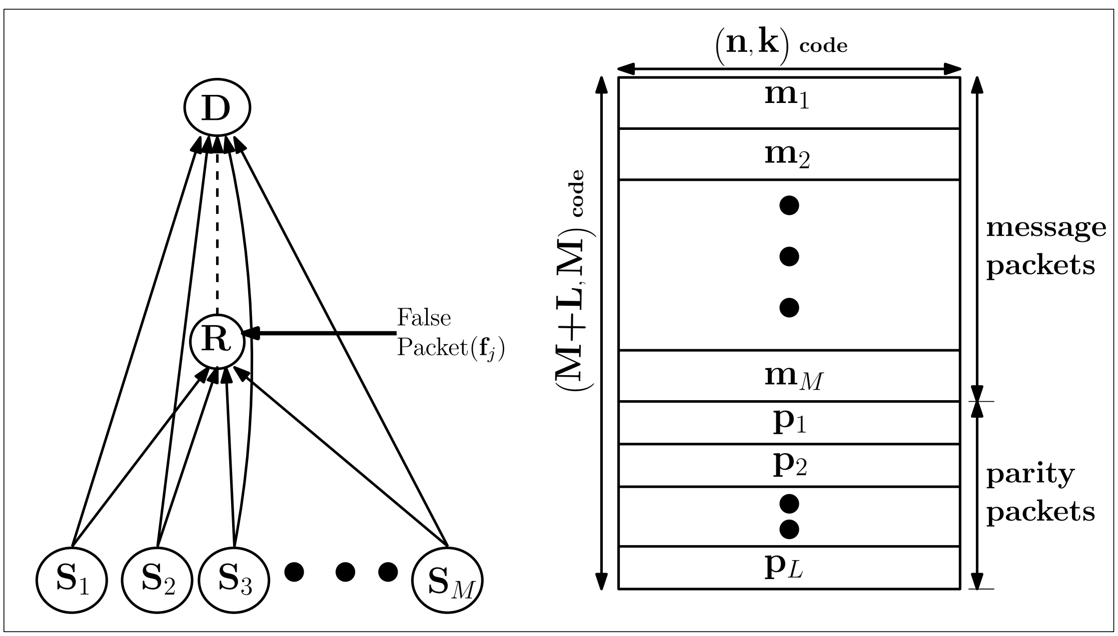

Figure 3.1 System Model for false injection

We consider a two-hop multi-access relay network composed of M sources S_i, $i \in \{1,2,...,$ $M\}$, one destination D and one relay R as shown in Fig. 3.1. In first phase, each source generates an independent message packet $\mathbf{m}_i$ using a (n, k) linear block code, denoted by $\mathcal{C}_r$, where n is the code length and k is the number of information bits in the codeword. We assume that the source-relay and relay-destination links are error free after channel decoding. Each source transmits $\mathbf{m}_i$ to the relay and destination on orthogonal channel (time or frequency) using BPSK modulation. The relay stores the received packets in an $M \times n$ array, where the i^{th} row is $\mathbf{m}_i$. Then it generates column wise parities using a $(M + L, M)$ linear block code, denoted by $\mathcal{C}_c$, such that each column is a valid linear block codeword where $M + L$ is the codeword length in column direction. The j^{th} parity packet can be expressed as

$$\mathbf{p}_j = \sum_{i=1}^{M} a_{ij}\mathbf{m}_i, \quad j \in \{1, 2, ..., L\} \tag{3.1}$$

for some coefficients $\{a_{ij}\} \in GF(2)$ and all the operations are in $GF(2)$. In second phase, L parity packets are transmitted to the destination.

Attack Model: We assume that the relay inserts a different randomly chosen valid codeword in $n_c - k_c$ rows of parity bits with probability p. We assume that the adversary can access the relay and modify the parity packets by inserting a non-zero vector $\mathbf{f}_j$ in $\mathbf{p}_j$

$$\mathbf{p}'_j = \sum_{i=1}^{M} a_{ij}\mathbf{m}_i + \mathbf{f}_j, \quad j \in \{1, 2, ..., L\} \tag{3.2}$$

It is of interest to the attacker to choose $\mathbf{f}_j$ to be a valid row codeword to avoid detection [56]. Subsequently we will assume $\mathbf{f}_j$ is a valid codeword. We assume semi-malicious behavior by the adversary because it will try to mask its malicious behavior. Hence $\mathbf{f}_j$ is a non-zero, valid row codeword with probability p_f.

Let $\mathcal{M}(\mathbf{m}_i)$ denote binary phase shift key (BPSK) modulation operation on a vector $\mathbf{m}_i$ where $\mathcal{M}(\mathbf{m}_i) = (-1)^{m_{ik}}$ and $m_{ik} \in \{0, 1\}$ is the k^{th} bit of $\mathbf{m}_i$, k = 1,2,...,n. Let

$$\begin{aligned} \mathbf{y}_{si} &= \mathbf{g}_{si}\sqrt{E_i}\mathcal{M}(\mathbf{m}_i) + \mathbf{n}_{si}, \quad i \in \{1, 2, ..., M\} \\ \mathbf{y}_{rj} &= \mathbf{g}_{rj}\sqrt{E_r}\mathcal{M}(\mathbf{p}'_j) + \mathbf{n}_{rj}, \quad j \in \{1, 2, ..., L\} \end{aligned} \tag{3.3}$$

denote the received signals from S_i and R, respectively, at the destination where $\mathbf{g}_{si}$ is the channel gain vector between the i^{th} source and the destination and $\mathbf{g}_{rj}$ is the channel gain vector between the relay and destination when transmitting the j^{th} parity packet. The elements of channel gain vector are independent complex Gaussian random variable with mean 0 and variance $d_{id}^{-\alpha}$ denoted by $\mathcal{CN}(0, d_{id}^{-\alpha})$ where α is the path loss exponent and d_{id} is the distance between the i^{th} source and the destination. Similarly the elements of channel gain vector, $\mathbf{g}_{rj}$, are independent $\mathcal{CN}\left(0, d_r^{-\alpha}\right)$ where d_r is the distance between the relay and destination. $\mathbf{n}_{si}$, $\mathbf{n}_{rj}$ are the noise vectors, modeled as $\mathcal{CN}(0, N_0)$. E_i and E_r are the transmit symbol energies from S_i and R, respectively.

The destination receives the codewords transmitted from the sources and the relay and stores them in an $(M + L) \times n$ array to form a linear block product code. It then uses the product decoding algorithm proposed by Pyndiah in [45] to decode the source messages.

3.3 Chase-Pyndiah Decoding Algorithm

In this section we briefly describe Chase-Pyndiah decoding algorithm for the iterative decoding of product codes. The details can be found in [45]. The decoding is performed on rows and columns alternately. One full iteration is said to be completed when all the rows and then all the columns are decoded and a half iteration is said to be completed when either all the rows or all the columns are decoded. We assume that the rows are decoded first.

Let I_{ik}^t denote the soft input at the beginning of the t^{th} half iteration for k^{th} bit of the i^{th} source packet, $i = 1, 2, ..., M$, $k \in \{1, 2, ..., n\}$ and I_{jk}^t denote the soft input for the j^{th} parity packet where $j \in \{1, 2, ..., L\}$. For odd values of t, row decoding is performed whereas for even values of t, column decoding is performed.

In the first half iteration, the soft input to the decoder is the normalized channel log likelihood ratio (LLR) for each bit:

$$\begin{aligned} I_{ik}^1 &= \Re e\{g_{sik}^* y_{sik}\} \\ I_{jk}^1 &= \Re e\{g_{rjk}^* y_{rjk}\} \end{aligned} \tag{3.4}$$

where g_{sik} is the k^{th} element of channel gain vector and y_{sik} is the k^{th} received bit from i^{th} source node, g_{rjk} is the k^{th} element of channel gain vector and y_{rjk} is the k^{th} received bit of j^{th} parity packet from the relay. Correspondingly let $\mathbf{I}_i^t$ denote the soft input row vector for i^{th} source packet and $\mathbf{I}_j^t$ denote the soft input row vector for j^{th} parity row at the beginning of the t^{th} half iteration. The extrinsic information is calculated after every half iteration for each bit and added to initial soft input to generate updated soft input for next half iteration.

Let $\mathbf{W}$ denote the set containing erasure bit positions. If there are no positions to be erased, then $\mathbf{W} = \{\phi\}$ i.e. it is an empty set. In our proposed scheme, $\mathbf{W}$ may be non-empty during column decoding only. We erase the j^{th} bit position if the false injection is detected and cannot be removed successfully from j^{th} parity packet.

Following are the main steps of the algorithm for soft decoding of particular row at the t^{th} half iteration, $t \in \{1, 2, 3, ..., t_{max}\}$ where t_{max} is the maximum number of half iterations. After all the rows are decoded and then similar steps are followed for the column decoding also.

1. Let $\mathbf{y}$ denote the received row vector, $\mathbf{y} \in \{\mathbf{y}_{si}, \mathbf{y}_{rj}\}$ for $i \in \{1, 2, ..., M\}$, $j \in \{1, 2, ..., L\}$. Let $\mathbf{I}^t$ denote the soft input row vector, $\mathbf{I}^t \in \{\mathbf{I}_i^t, \mathbf{I}_j^t\}$. First $2^{\lfloor d_{min}/2 \rfloor}$ binary test sequences are constructed as outlined below, where d_{min} is the minimum distance of $\mathcal{C}_r$.
   (a) The positions of $\lfloor d_{min}/2 \rfloor$ least reliable bits of $\mathbf{I}^t$ are determined. The reliability of k^{th} bit is defined as the absolute value of it's LLRs, I_{jk}^t and I_{ik}^t. For determining least reliable bits, erasure bit positions in $\mathbf{W}$ are not taken into consideration.
   (b) Let $\mathcal{M}^{-1}(\mathbf{I}^t) \in GF(2)$ denote the hard decision on $\mathbf{I}^t$ and $\mathcal{M}^{-1}(\mathbf{y}) \in GF(2)$ denote the hard decision on $\mathbf{y}$.
   (c) Next a test pattern set, $\mathbf{T} = \{\mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_{2^{\lfloor d_{min}/2 \rfloor}}\}$, is formed, which consists of the vectors with a single "1" in the least reliable positions and "0" in the other positions, two "1"s at least reliable positions and "0" at other positions, ..., $\lfloor d_{min}/2 \rfloor$ "1"s at the least reliable positions and "0" in other positions.
   (d) Next a test sequence set, $\mathbf{U} = \{\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{2^{\lfloor d_{min}/2 \rfloor}}\}$ is formed, where $\mathbf{u}_q = \mathcal{M}^{-1}(\mathbf{I}^t) \oplus \mathbf{t}_q$, $q = 1, 2, ..., 2^{\lfloor d_{min,r}/2 \rfloor}$, and $\oplus$ denotes bit wise XOR in $GF(2)$.
   (e) During column decoding, for each sequence $\mathbf{u}_q$, place zeros at all erasure positions and label it as $\mathbf{u}_q^0$ and then place ones at all erasure positions and label it as $\mathbf{u}_q^1$. Amongst $\mathbf{u}_q^0$ and $\mathbf{u}_q^1$, the one at smaller Hamming distance to $\mathcal{M}^{-1}(\mathbf{y})$ is selected and labeled as $\mathbf{z}_q$. If $\mathbf{W}$ is an empty set, then $\mathbf{z}_q = \mathbf{u}_q$. During row decoding since no erasures are generated hence $\mathbf{z}_q = \mathbf{u}_q$.
   (f) $\mathbf{z}_q$ is decoded using algebraic decoder to produce a codeword $\mathbf{c}_q \in \mathcal{C}_r$. Let $\Omega^t =$

$\{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_{2^{\lfloor d_{min}/2 \rfloor}}\}$ denote the set of candidate codewords at the t^{th} half iteration.

2. Let

$$\mathbf{c}^t = \arg \min_{\mathbf{c}_q \in \Omega^t} ||\mathbf{y} - \mathbf{g}\mathcal{M}(\mathbf{c}_q)||^2 \tag{3.5}$$

where $\mathbf{g} \in \{\mathbf{g}_{si}, \mathbf{g}_{rj}\}$ and let

$$\underline{\mathbf{c}}_k^t = \arg \min_{\underline{\mathbf{c}}_q \in \Omega^t - \mathbf{c}^t} ||\mathbf{y} - \mathbf{g}\mathcal{M}(\underline{\mathbf{c}}_q)||^2 \tag{3.6}$$

where the k^{th} bit of $\underline{\mathbf{c}}_k^t$ is different from that of $\mathbf{c}^t$. Note that $\mathcal{M}(\mathbf{c}^t)$ is at the minimum Euclidean distance from $\mathbf{y}$. $\underline{\mathbf{c}}_k^t$ is called competing codeword. While processing a particular row or column, $\mathbf{c}^t$ is same for all the bits of corresponding row or column whereas $\underline{\mathbf{c}}_k^t$ may or may not be same for all the bits.

3. The new LLR of the k^{th} bit of $\mathbf{c}^t$ is given by

$$L_k^t = \left(\frac{||\mathbf{y} - \mathbf{g}\mathcal{M}(\underline{\mathbf{c}}_k^t)||^2 - ||\mathbf{y} - \mathbf{g}\mathcal{M}(\mathbf{c}^t)||^2}{4} \right) \mathcal{M}(c_k^t) \tag{3.7}$$

where c_k^t is the k^{th} bit of $\mathbf{c}^t$. In some cases it is not possible to find competing codeword $\underline{\mathbf{c}}_k^t$. Then, the LLR for k^{th} bit is estimated as

$$L_k^t = \beta \times \mathcal{M}(c_k^t) \tag{3.8}$$

where $\beta \geq 0$ is a constant which can be pre-calculated as in [45] or adaptively calculated as in [46].

4. The extrinsic information for k^{th} bit is calculated as

$$\begin{aligned} E_{ik}^t &= L_k^t - I_{ik}^t \quad \text{and} \\ E_{jk}^t &= L_k^t - I_{jk}^t \end{aligned} \tag{3.9}$$

when i^{th} source packet and j^{th} parity packet from relay is being processed.

5. The extrinsic information for all the bits of the product code are calculated and stored in a matrix. Then the absolute mean of the extrinsic information is calculated as

$$\mathcal{E}^t = \frac{\sum_i \sum_k |E_{ik}^t| + \sum_j \sum_k |E_{jk}^t|}{(M + L) \times n} \tag{3.10}$$

where $|.|$ denotes the absolute value.

6. The extrinsic information for k^{th} bit is normalized by absolute mean calculated to produce

$$\overline{E}_{ik}^{t} = \frac{E_{ik}^{t}}{\mathcal{E}^{t}} \quad \text{and}$$
$$\overline{E}_{jk}^{t} = \frac{E_{jk}^{t}}{\mathcal{E}^{t}} \tag{3.11}$$

7. The normalized extrinsic information is then added to the soft input to produce updated soft input for next half iteration

$$I_{ik}^{t+1} = I_{ik}^{t} + \alpha \times \overline{E}_{ik}^{t}$$
$$I_{jk}^{t+1} = I_{jk}^{t} + \alpha \times \overline{E}_{jk}^{t} \tag{3.12}$$

   where α is a constant scale factor increased from 0 to 1 as iterations increase to control the effect of extrinsic information on decoding. It can be pre-calculated as in [45] or adaptively calculated as in [46].

8. Once all the rows are processed, similar decoding is performed on all the columns. This completes one full iteration.

9. At the end of each full iteration, hard decision is made on the LLRs, L_k^t, for all the rows $i = 1, 2, ..., M$ and $j = 1, 2, ..., L$. Then they are decoded using algebraic decoder for $\mathcal{C}_r$ block code. Subsequently column decoding is performed using algebraic decoder for $\mathcal{C}_c$ block code on each column, $k = 1, 2, ..., n$, to obtain final $(M + L) \times n$ decoded bits.

10. The above process is repeated for a predetermined maximum number of iterations to obtain final decision on source and relay parity packets.

3.4 Iterative Packet Recycling

In this section we will describe the iterative packet recycling scheme. It consists of detecting and removing false injection detection. We will describe two approaches to the detection, first is physical-layer based and second is signature based approach. The basic idea in both approaches is to use noisy packets received from sources to remove the false injection. The packet recycling scheme is performed at the destination while performing row decoding in each iteration. In first approach, the false injection is detected solely based on received packets from the sources, while in the second scheme false injection is detected based on signatures received from sources.

3.4.1 Detection of Polluted Packet

Let

$$\mathbf{m}_i^t = \mathbf{m}_i + \mathbf{e}_i^t \quad i \in \{1, 2, ..., M\} \tag{3.13}$$

denote the decoded source packet for $\mathbf{m}_i$ at t^{th} iteration and $\mathbf{e}_i^t$ denotes the error vector after decoding. For detecting false injection, we assume that t is odd, therefore row decoding is performed at t^{th} iteration and column decoding is performed at $t+1^{th}$ iteration.

Assuming that all possible $\mathbf{f}_j \in \mathcal{C}_r$ are equi-probable, the false injection $\mathbf{f}_j$ can be detected using maximum likelihood(ML) decoding rule as:

$$\mathbf{f}_j^t = \arg\max_{\mathbf{f}_j \in \mathcal{C}_r} P\left(\hat{\mathbf{p}'}_j^t | \mathbf{f}_j, \mathbf{m}_1^t, \mathbf{m}_2^t ....., \mathbf{m}_M^t\right) \tag{3.14}$$

where $\mathbf{p'}_j^t$ is the decoded codeword for $\mathbf{p}'_j$ at the t^{th} iteration.

$$\mathbf{p'}_j^t = \mathbf{p}'_j + \mathbf{e}_j^t \quad j \in \{1, 2, ..., L\} \tag{3.15}$$

From [57], it can be simplified to

$$\mathbf{f}_j^t = \arg\min_{\mathbf{f}_j \in \mathcal{C}_r} d_H(\hat{\mathbf{p}'}_j^t, \mathbf{f}_j) \tag{3.16}$$

where

$$\begin{aligned} \hat{\mathbf{p}'}_j^t &= \mathbf{p}'_j - \sum_{i=1}^{M} a_{ij}\mathbf{m}_i^t \\ &= \mathbf{f}_j - \sum_{i=1}^{M} a_{ij}\mathbf{e}_i^t + \mathbf{e}_j^t \end{aligned} \tag{3.17}$$

and $d_H(\hat{\mathbf{p}'}_j^t, \mathbf{f}_j)$ denotes the Hamming distance between $\hat{\mathbf{p}'}_j^t$ and $\mathbf{f}_j$. If $\mathbf{f}_j^t = 0$, the j^{th} parity packet is *determined* to be unpolluted.

3.4.2 Removal of Polluted Packet

After the estimation of false injection, the destination subtracts $\hat{\mathbf{f}}_j^t$ from the parity packet $\mathbf{p}'_j$ received from the relay to get a recycled packet:

$$\begin{aligned} \tilde{\mathbf{p}}_j^t &= \mathbf{p'}_j^t - \mathbf{f}_j^t \\ &= \sum_{i=1}^{M} a_{ij}\mathbf{m}_i + \mathbf{f}_j - \mathbf{f}_j^t \end{aligned} \tag{3.18}$$

3.4.3 Iterative Detection and Mitigation of False Injection

The detection and mitigation of false injection can be done iteratively. If $\mathbf{f}_j^t = 0$, then the LLRs, L_{jk}^t in Eq. (3.7) are not changed. However, if $\mathbf{f}_j^t \neq 0$, the soft input of the j^{th} parity packet is changed to

$$I_{jk}^t \leftarrow I_{jk}^t \times \mathcal{M}(f_{jk}^t) \quad \forall\, k \in \{1, 2, ..., n\} \tag{3.19}$$

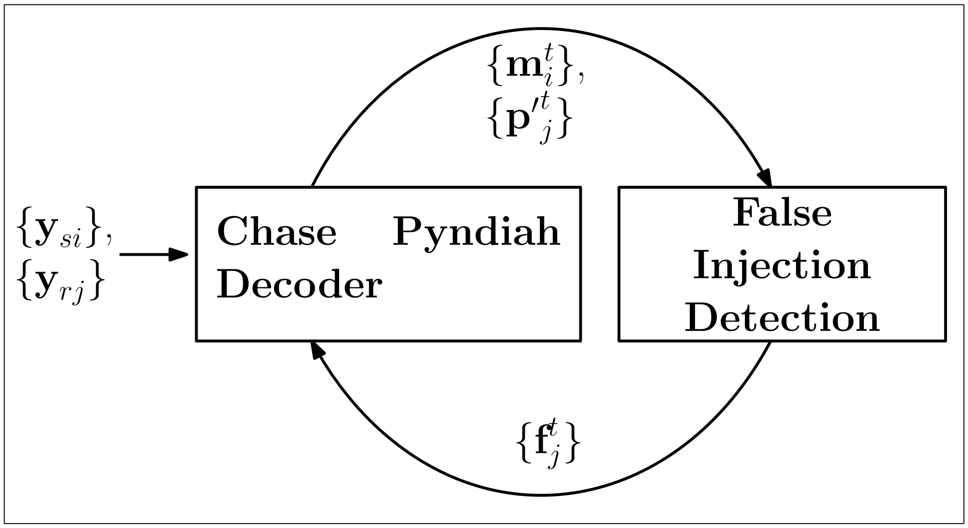

Figure 3.2 Iterative detection and mitigation of false injection

The j^{th} parity row is processed again from step 1 of the Chase Pyndiah algorithm after this change. This multiplication effectively removes the detected false packet since at all those k^{th} bit positions where $f_{jk}^t = 1$, $\mathcal{M}(f_{jk}^t) = -1$ and hence it essentially inverts the sign of LLR. This inversion results in changing of $0 \to 1$ and $1 \to 0$ in GF(2) domain. This iterative process is shown in Fig.3.2.

3.4.4 Recycling without Signatures

In this scheme all parity rows (packets) need to be inspected using the scheme described above. We propose two approaches, in the first approach, the destination uses *decoded* source packets $\mathbf{m}_i$, $i = 1, 2, ..., M$ to estimate false injection. The error performance of this approach, as shown in Fig.3.4, is similar to the case when the relay parities are completely discarded and the source packets are decoded from data received via source - destination link only. This is because source - destination link is unreliable and the estimation error probability of falsely injected packet is as good as source - destination link. This results in high false alarm and therefore the overall performance of the decoding is poor.

In second approach, the destination uses *demodulated* instead of *decoded* source packets to estimate false injection. The reason to consider demodulated approach is because in iterative decoding, the overhead to decode all the source packets repeatedly in order to estimate false injection can be substantial for long block lengths and it might cause delay at the destination.

Let $\bar{\mathbf{m}}_i$ and $\bar{\mathbf{p}}_j$ denote the demodulated row packet in GF(2) from source S_i and j^{th} parity row packet from the relay respectively, $\bar{\mathbf{m}}_i = \mathcal{M}^{-1}(\mathbf{y}_i)$, and $\bar{\mathbf{p}}_j = \mathcal{M}^{-1}(\mathbf{y}_{rj})$ where $\mathcal{M}^{-1}(.)$ denotes demodulation operation.

$$\begin{aligned} \bar{\mathbf{m}}_i &= \mathbf{m}_i + \bar{\mathbf{e}}_i \\ \bar{\mathbf{p}}_j &= \mathbf{p}_j + \mathbf{e}_{rj} + \mathbf{f}_j \end{aligned} \tag{3.20}$$

where $\bar{\mathbf{e}}_i$ denotes the error vector after demodulation $\mathbf{m}_i$. The maximum likelihood(ML) rule to decode, assuming all possible $\mathbf{f}_j \in \mathcal{C}_r$ to be equi-probable, can be stated as

$$\hat{\mathbf{f}}_j = \arg\max_{\mathbf{f}_j \in \mathcal{C}_r} P\left(\bar{\mathbf{p}}_j | \mathbf{f}_j, \bar{\mathbf{m}}_1, \bar{\mathbf{m}}_2 ....., \bar{\mathbf{m}}_M\right) \tag{3.21}$$

This can be further simplified as

$$\hat{\mathbf{f}}_j = \arg\min_{\mathbf{f}_j \in \mathcal{C}_r} d_H(\bar{\mathbf{p}}'_j, \mathbf{f}_j) \tag{3.22}$$

where

$$\bar{\mathbf{p}}'_j = \bar{\mathbf{p}}_j - \sum_{i=1}^{M} a_{ij} \bar{\mathbf{m}}_i + \mathbf{e}_{rj} \tag{3.23}$$

3.4.5 Signature aided Recycling Scheme

In this subsection we discuss the signature-aided recycling scheme. Signatures are used to check the authenticity of the received message [58; 59; 60]. Let $\mathbf{h}_i = \mathcal{H}(\mathbf{m}_i)$ denote the signature of $\mathbf{m}_i$, where $\mathcal{H}(.)$ is the hash function, and $\mathbf{h}_{rj} = \mathcal{H}(\mathbf{p}'_j)$ denote the signature of j^{th} parity packet $\mathbf{p}'_j$. We assume that the destination receives the signatures without error. Using the signatures $\mathbf{h}_1$, $\mathbf{h}_2$,...,$\mathbf{h}_M$, the signature for the j^{th} parity packet can be generated by the destination:

$$\hat{\mathbf{h}}_{rj} = \mathcal{F}(\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_M) \tag{3.24}$$

where $\mathcal{F}(.)$ is the function to generate hash value of network coded packet[58; 59; 60]. Then, $\mathbf{h}_{rj}$ received from the relay is compared with $\hat{\mathbf{h}}_{rj}$. If $\hat{\mathbf{h}}_{rj} = \mathbf{h}_{rj}$, then normal iterative decoding is performed. If they do not match then the j^{th} row is marked as polluted and packet recycling procedure, as described in Section IV A, B and C, is performed only on the marked rows. In order to check if the recycling process has yielded correct parity packet or not, the destination generates the signature of recycled packet, $\tilde{\mathbf{h}}_{rj} = \mathcal{H}(\tilde{\mathbf{p}}_j^t)$ and compares $\tilde{\mathbf{h}}_{rj}$ with $\hat{\mathbf{h}}_{rj}$. If they match, then the updated soft LLRs given by Eq.(3.12) for j^{th} parity row are used in column decoding during $t+1^{th}$ half iteration. If they do not match, then the bit position corresponding to j^{th} parity row is erased during the column decoding by adding the j^{th} bit position in erasure set $\mathbf{W}$. This step is called *signature based integrity check* of recycled packet. The algorithm for this scheme has been outlined in Fig.3.3. It will be shown that the signature aided recycling scheme outperforms the recycling without signatures.

3.4.6 Traditional scheme

The traditional scheme is after detecting the false injection using signatures, all polluted parity packets are discarded. Hence erasures are created on the corresponding rows[54; 55].

3.5 Simulation Results

In this section we will present simulation results. We assume there are 7 sources ($M = 7$), each transmitting (15,7) binary BCH codeword and are equidistant from the destination. The source-destination distance is normalized to one i.e. $d_{id} = 1$. We assume error free source-relay and relay-destination channel. The relay, after overhearing all the codewords, sends parity packets to the destination, each generated from (15,7) BCH code. Hence (15,7)×(15,7) product code is formed at the destination. We assume Rayleigh fading with additive while Gaussian noise and path loss exponent, $\alpha = 4$.

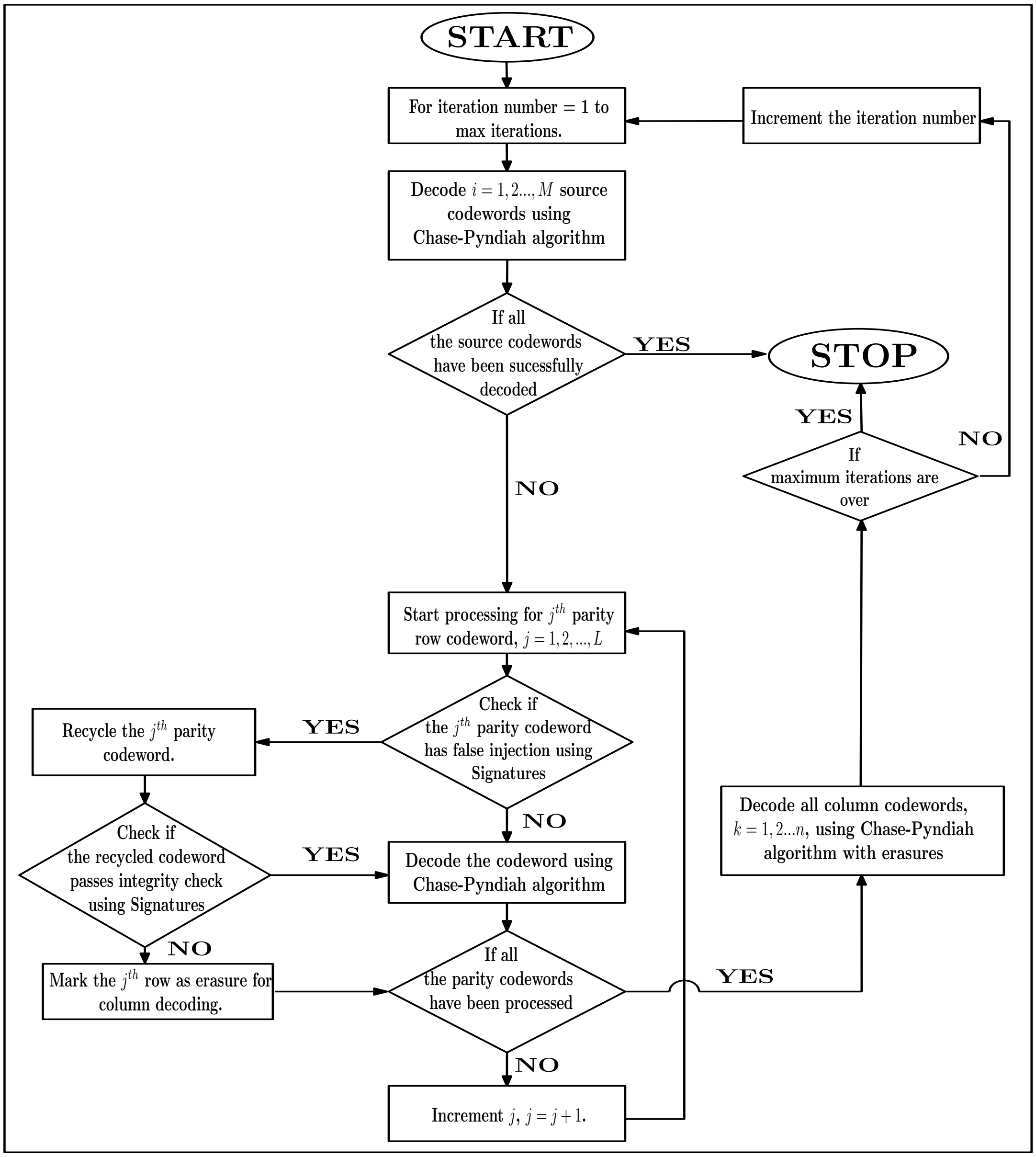

Figure 3.3 Signature based recycling with integrity check

Fig.3.4 shows performance comparison for various techniques. The probability of false injection p_f is set to 0.3. The figure shows approximate maximum likelihood decoding performance for (15,7) BCH code which is attained by sending the codeword via direct source-to-destination link. This is the case when destination completely discards the parity packets from relay and is denoted by "No Network Coding". The probability of codeword error is given by [72; 73; 74]

$$P_E \leq \sum_{d=d_{min}}^{N} \frac{A_d}{\pi} \int_0^{\pi/2} \left(\frac{1}{(E_c/N_0)\text{cosec}(\theta) + 1} \right)^d d\theta \tag{3.25}$$

where A_d is the number of codewords having the hamming weight of d, N is the block length and d_{min} is the minimum distance of the code. At high SNR, this bound can be approximated as

$$P_E \approx \frac{A_{d_{min}}}{\pi} \int_0^{\pi/2} \left(\frac{1}{(E_c/N_0)\text{cosec}(\theta) + 1} \right)^{d_{min}} d\theta \tag{3.26}$$

It can be seen that recycling without signature performs similar to the case when all the relay parities are completely discarded by the destination and the source packets are decoded based on the packets received via direct source-destination link only. This is because source-destination link is unreliable and the estimation error probability of falsely injected packet is as good as the reliability of source-destination link. This results in high probability of false alarm. The signature based recycling scheme outperforms the recycling without signatures, since the probability of false alarm is practically zero in the former. Also the signature based integrity check ensures that the erroneously recycled packets are discarded in the column decoding. It can also be seen that the traditional cryptographic scheme that does not recycle polluted packets performs worse. This is because the probability of erasure for traditional scheme, which is equal to probability of false injection, remains same for all SNRs. However, as SNR increases, source packets are received with higher reliability. Therefore false injection can be detected and removed much more efficiently. This reduces erasure probability for packet recycling scheme and improves its performance.

Fig.3.5 shows performance comparison of non-signature based recycling schemes. At high SNR, the scheme in which the destination uses decoded source packets to estimate false injection performs little better. This is because at high SNR, the decoded source packets will be error free with high probability whereas the demodulated source packets might still have errors. This

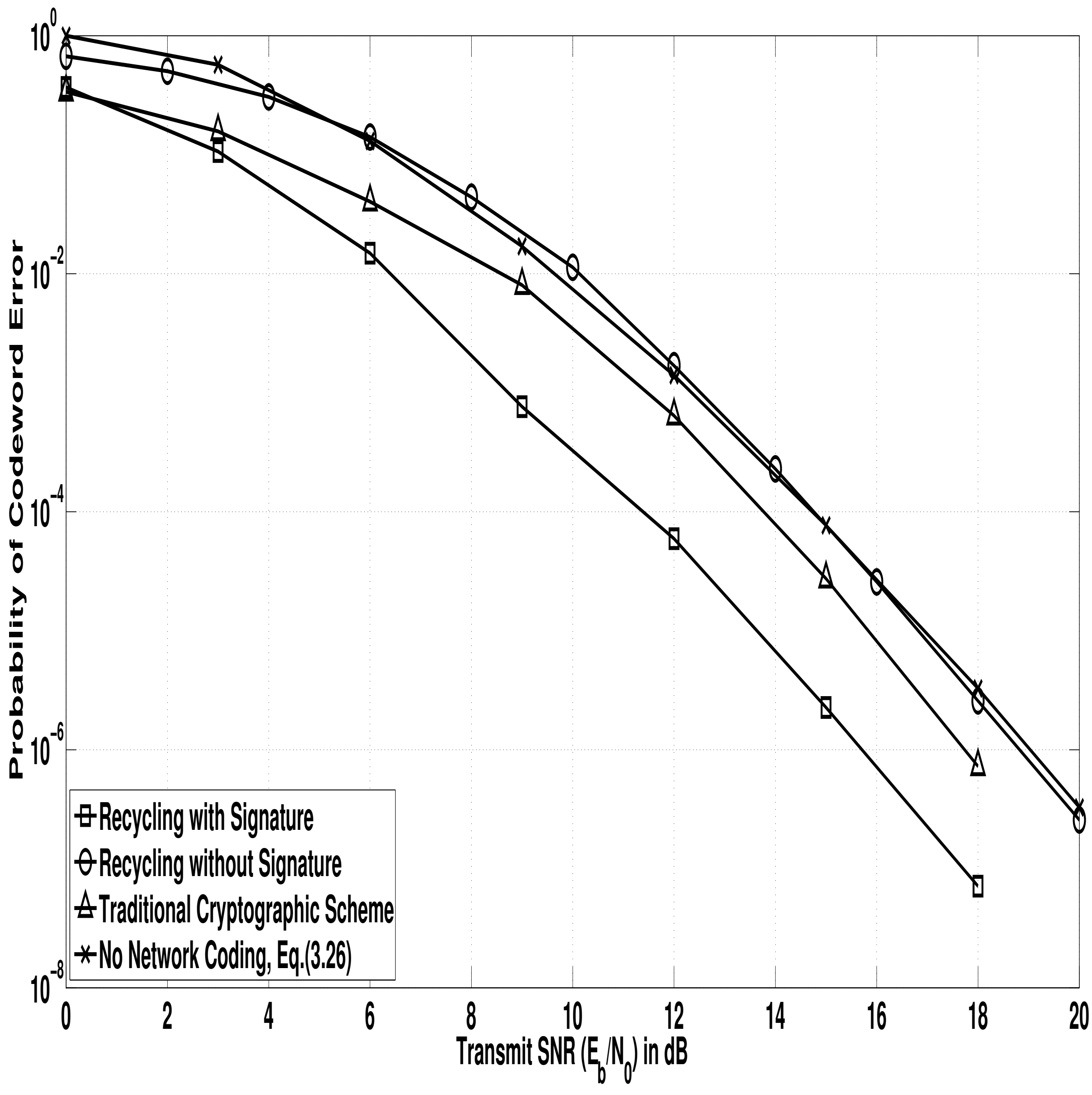

Figure 3.4 Performance comparison of various schemes for $p_f = 0.3$.

leads to an error in the detection and estimation of falsely injected packet and hence degrading the overall iterative decoding performance.

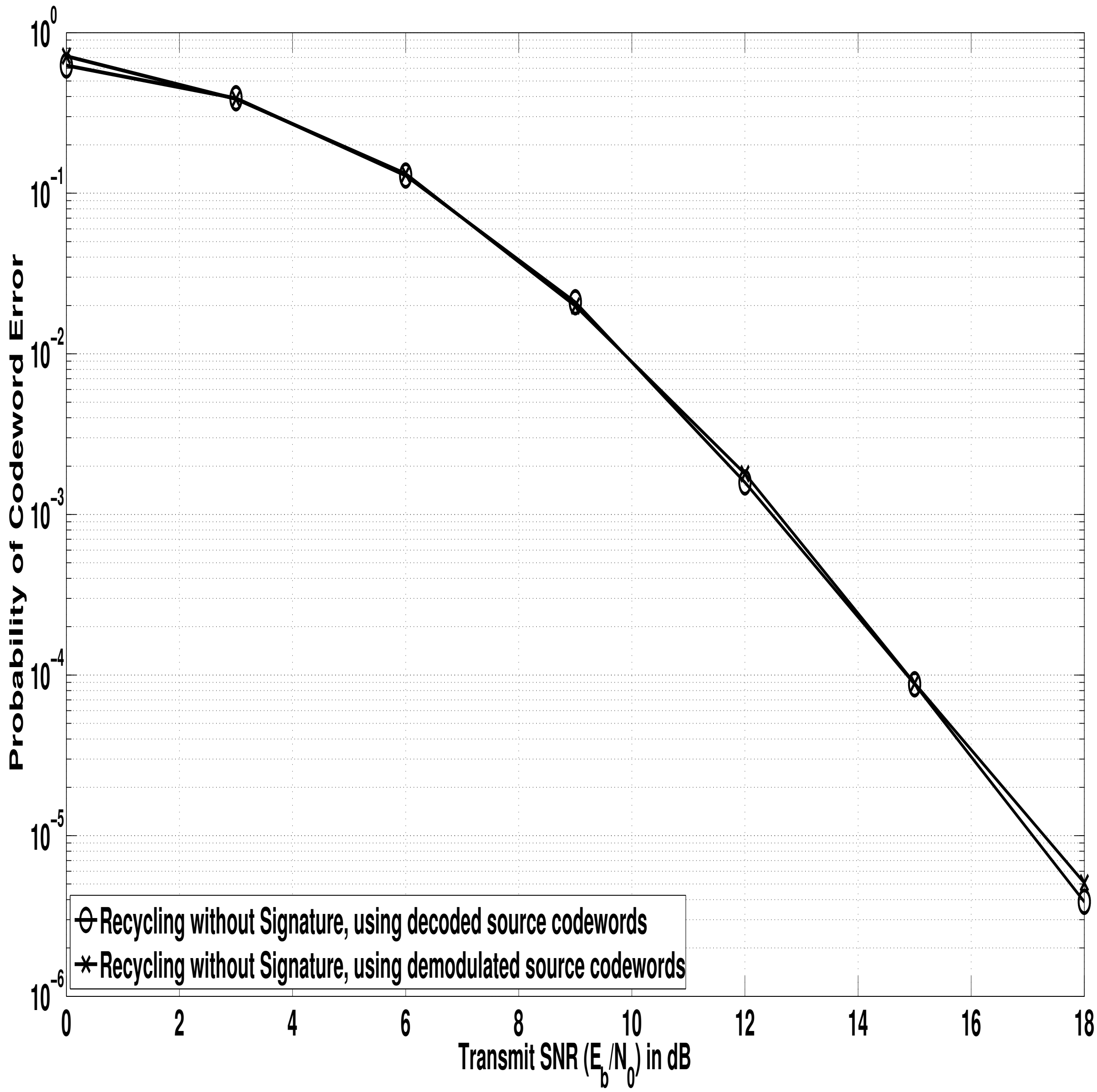

Figure 3.5 Performance comparison of Non Signature based schemes.

Fig.3.6 shows performance of signature based recycling scheme as a function of the number of decoding iterations. The probability of false injection, p_f, is set to 0.3. As expected, the decoding performance gets better with increasing number of iterations. The maximum gain in performance improvement is obtained by increasing number of iterations from one to two.

Increasing the number of iterations from two to three or four does improves performance, however the incremental improvement diminishes as the number of iterations increases. This is in line with other iterative decoding algorithms wherein maximum performance improvement is obtained by increasing iterations from one to two and further increase in number of iterations gives diminishing improvements.

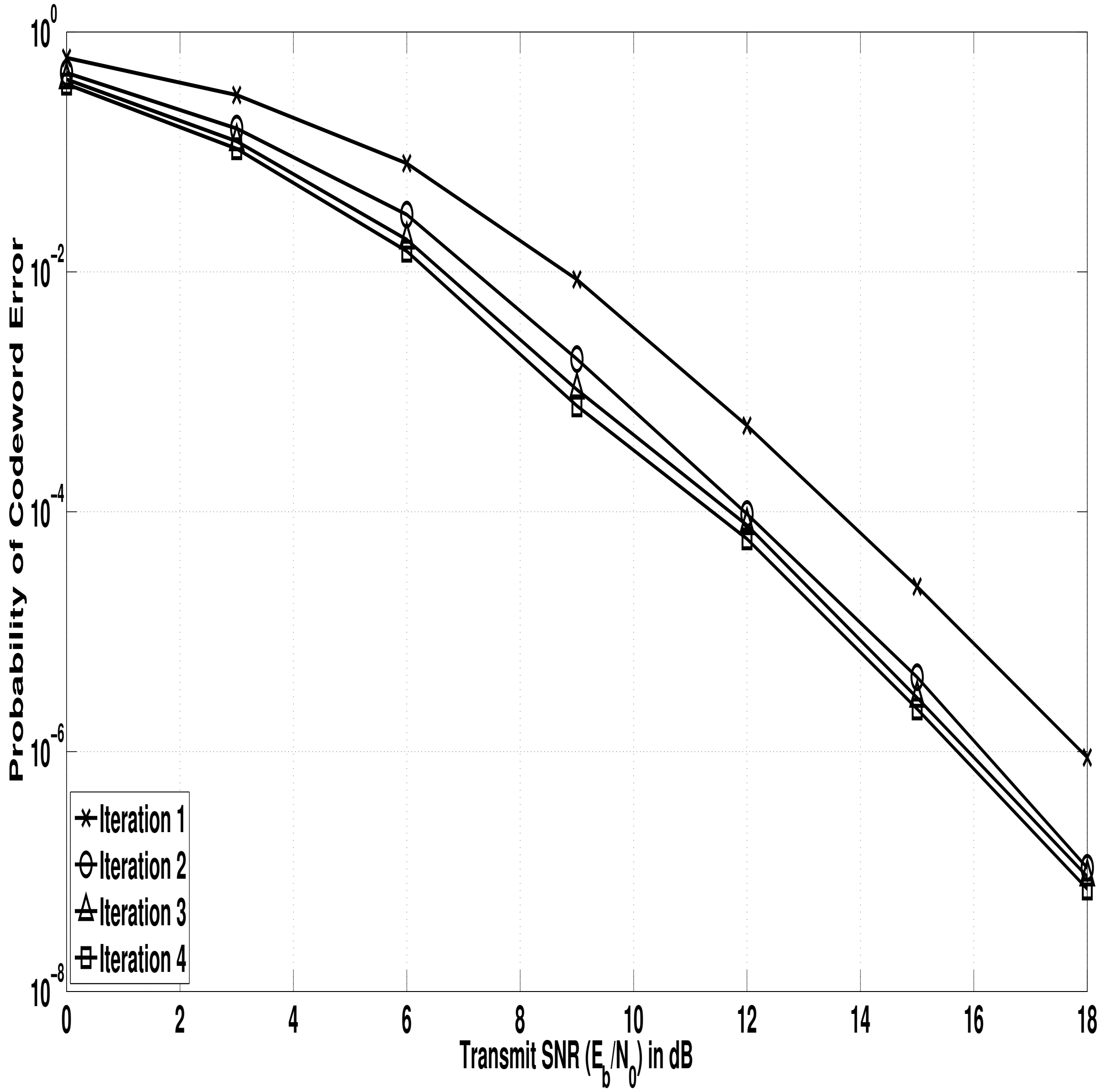

Figure 3.6 Performance of signature aided recycling scheme w.r.t number of iterations.

Fig.3.7 shows the performance variation of signature based recycling scheme vs probability of false injection, p_f. The number of decoding iterations is four. As expected, the decoding performance gets better with decreasing probability of false injection.

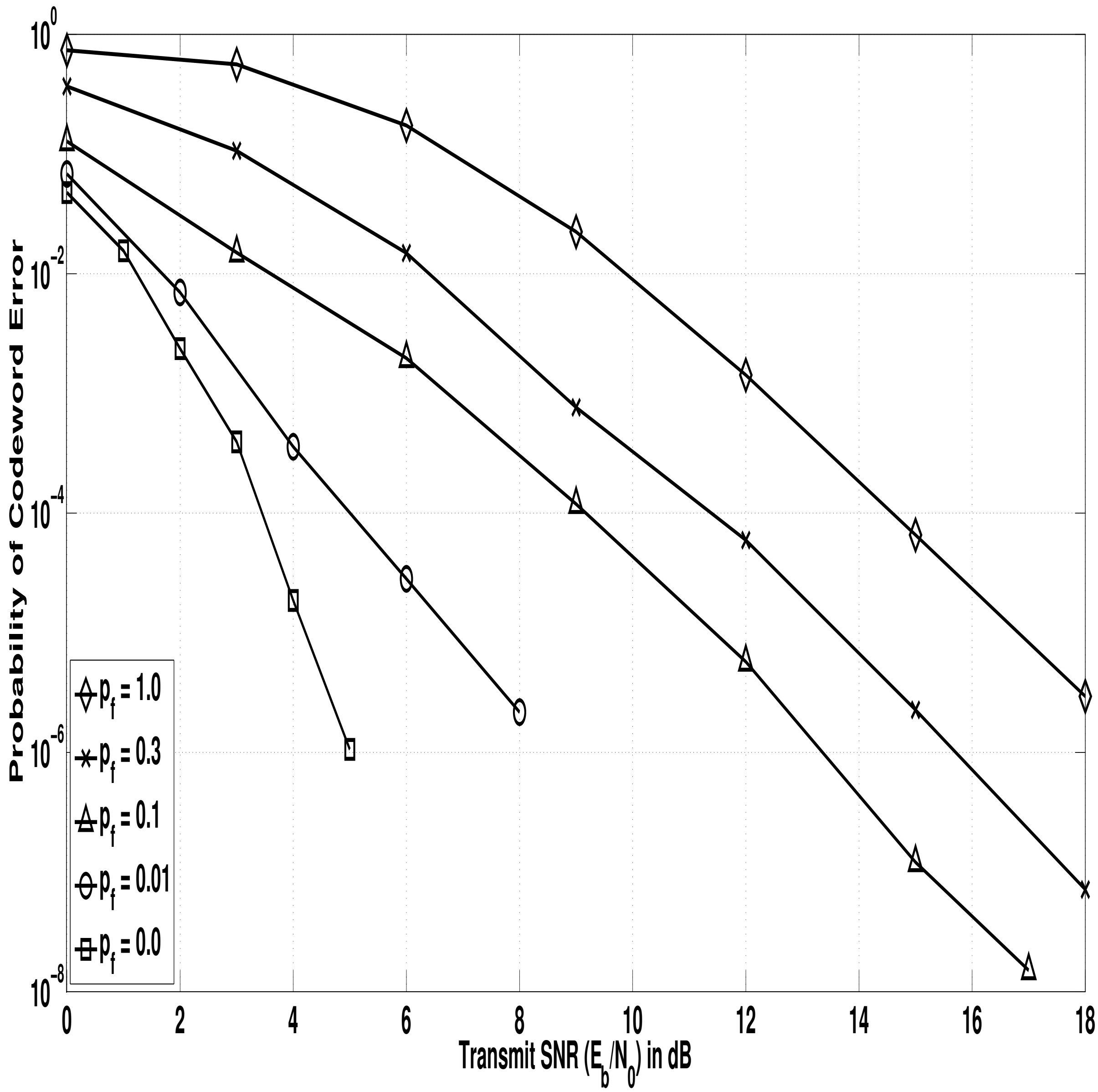

Figure 3.7 Performance variation of signature based scheme vs probability of false injection.

Fig.3.8 shows the probability of detection error $P(\mathbf{f}_j^t \neq \mathbf{f}_j)$ for packet recycling with signature and recycling without signature. It can be seen that, the detection of falsely injected packet becomes more accurate with iterations for the signature aided recycling scheme, while for recycling without signature, it remains the same as in first iteration. This improvement in the detection of false packet helps in improving the reliability of decoding as shown by Fig. 3.6.

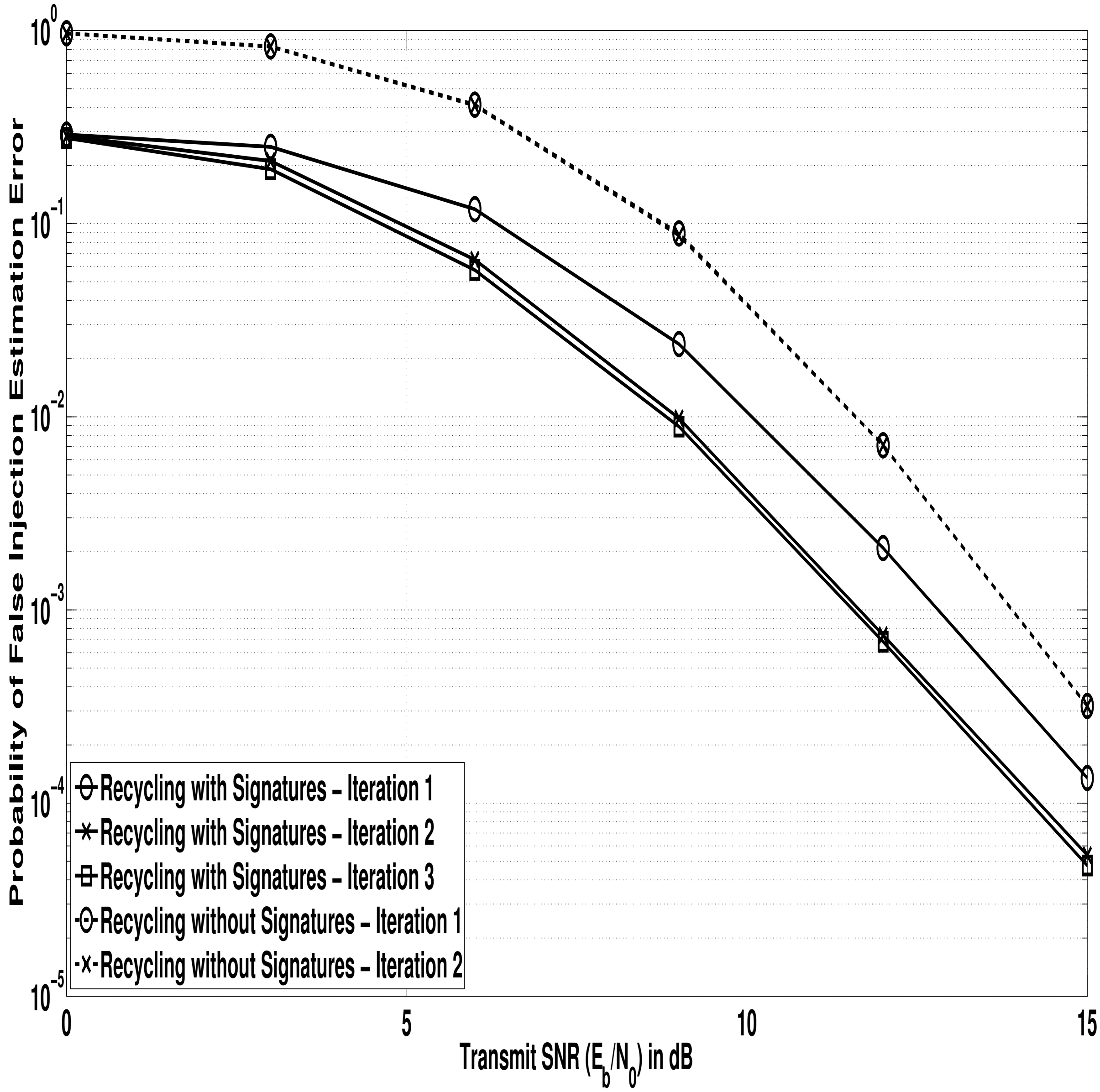

Figure 3.8 Probability of detection error, $P(\mathbf{f}_j^t \neq \mathbf{f}_j)$, versus SNR for different number of iterations

Fig.3.9 shows the performance comparison of traditional and proposed recycling scheme that employs signature. It can be seen that the proposed scheme outperforms the traditional scheme for all false injection probabilities.

We now assume there are 7 sources ($M = 7$), each transmitting (31,21) binary BCH codeword and are equidistant from the destination. The source-destination distance is normalized

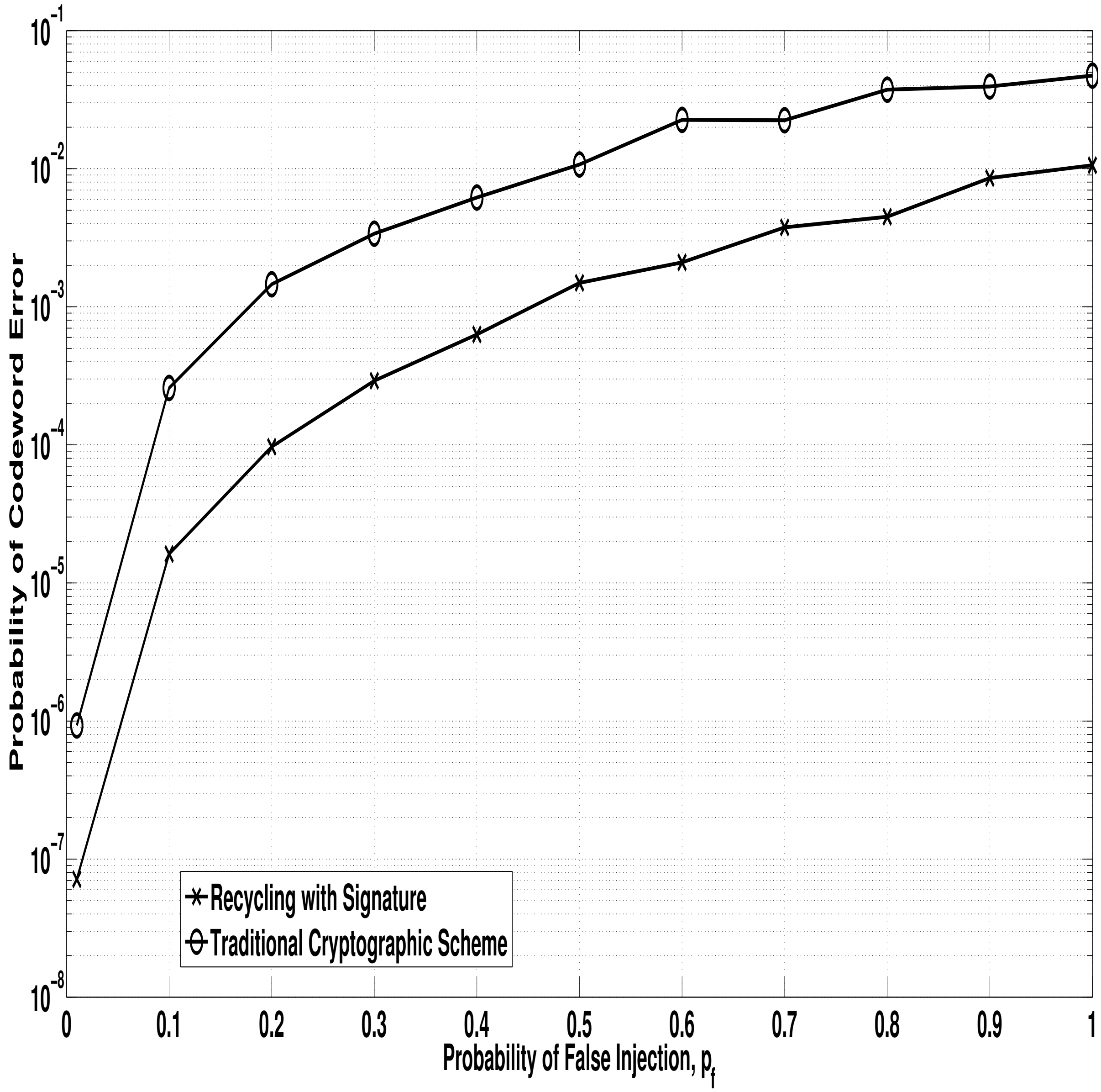

Figure 3.9 Performance Comparison of traditional and proposed scheme w.r.t probability of false injection, Transmit SNR $E_b/N_0 = 10$ dB

to one i.e. $d_{id} = 1$ The relay, after overhearing all the codewords, sends parity packets to the destination, each generated from (15,7) BCH code. Hence (31,21)×(15,7) product code is formed at the destination. We assume error free source-relay and relay-destination channel. We assume Rayleigh fading with additive while Gaussian noise and path loss exponent, $\alpha = 4$. Fig. 3.10 shows the performance comparison of traditional and proposed recycling scheme with signatures. It can be seen that the proposed scheme outperforms the traditional scheme in this case also.

3.6 Conclusion

In this work we proposed an iterative packet recycling technique that removes the falsely injected packet from the polluted packets and enhances the reliability of decoding at the destination. The proposed approach provides a significant improvement over the traditional approach that simply detects the presence of pollution attack and discards all polluted packets. Our future work is to derive the error probability when capacity approaching codes like LDPC codes are used.

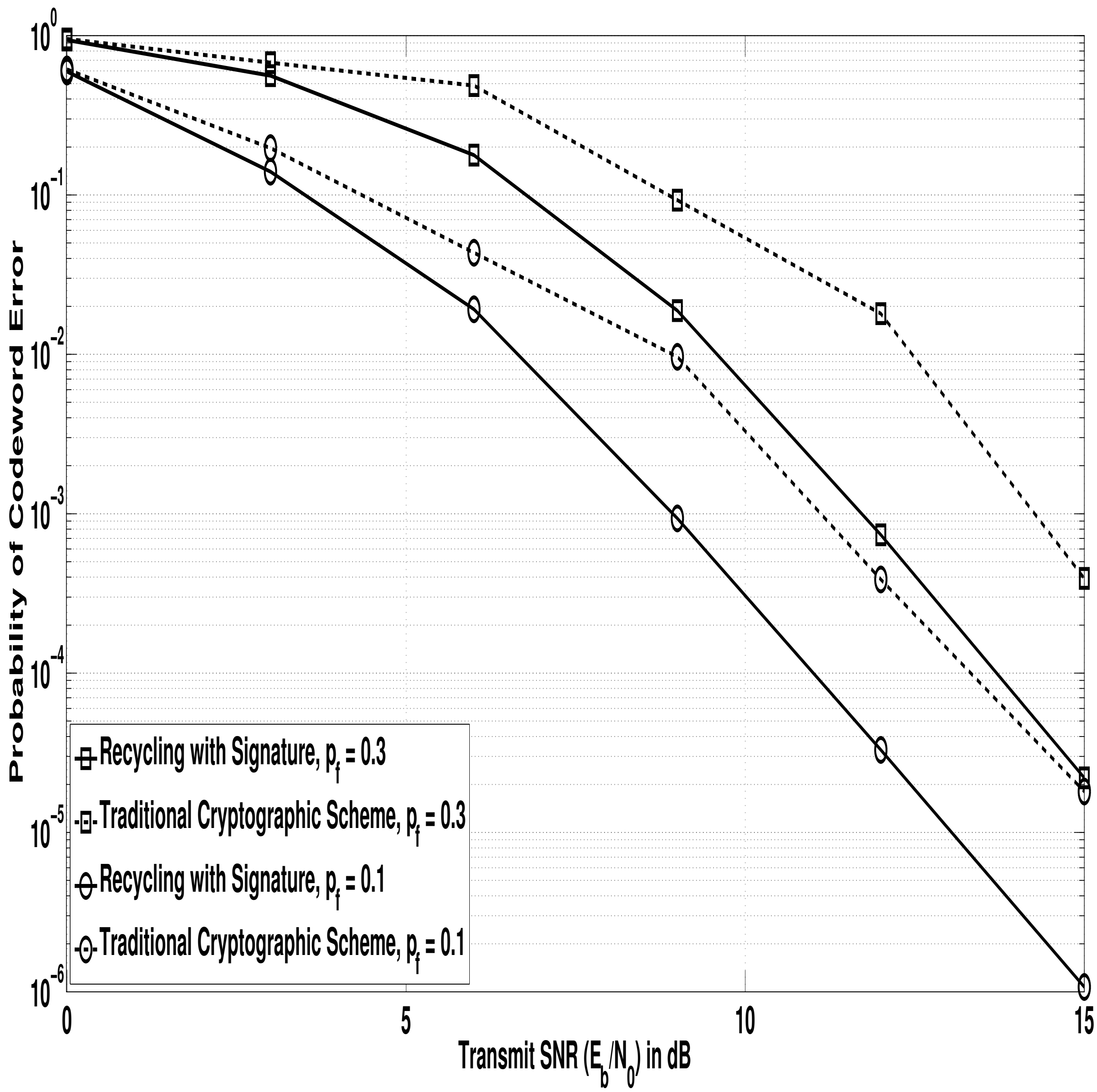

Figure 3.10 Performance comparison of traditional and proposed scheme for BCH (31,21) code for $p_f = 0.3$ and 0.1.

CHAPTER 4. RELAY SELECTION IN COOPERATIVE NETWORK CODING

4.1 Introduction

Cooperative communication has emerged as an important technology for satisfying growing demand of wireless services. In this scheme, one or more wireless nodes act as relay to facilitate communication between other node(s) and destination(s). A number of cooperative protocols and their variations have been proposed and analyzed in the literature, notable ones being Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CAF) [24; 25; 26; 7]. The central idea is that multiple relay nodes of the network cooperate to form a virtual antenna array. This exploits spatial diversity amongst cooperating nodes which improves the overall network performance. With increase in number of relays, the complexity associated with relaying increases since the relays generally have to transmit on orthogonal channels to avoid interference at the destination. The overcome this limitation, various relay selection (RS) criteria for amplify and forward, decode and forward, and their variations have been proposed in [33]. The most popular selection strategy is to select best relay which has best end to end performance in terms of capacity or signal to noise ratio at the destination[34], [38; 39; 40; 41; 42; 43; 44].

Network Coding (NC) is another efficient solution proposed to improve spectral efficiency for wired communication [8]. The intermediate relay nodes encode the incoming packets such that they can be reliably decoded at their destination. Due to broadcast nature of wireless medium, the intermediate relay nodes can overhear data from other source nodes. Hence network coding can be naturally combined with cooperative communication. Various network coded cooperation techniques have been proposed in the literature where the destination jointly decodes the information received from intended sources and relays[35; 36; 37].

In this work we consider design of relay selection with network coding in multiple access relay channel (MARC). In this scenario, multiple sources transmit data to common destination in orthogonal time/frequency slots in first phase. In second phase, a single relay is selected amongst relays to forward the network coded packet to destination. The destination jointly decodes the network coded packet along with the transmissions from the sources. We consider the case that a relay is allowed to participate in second phase only if it is able to decode all the sources correctly. This is done to avoid error propagation and as shown in our analysis, it maintains the maximum achievable diversity order. We propose a new relay selection scheme by taking source-destination channel links into account. The proposed scheme provides exactly same outage probability as the other state-of art relay selection schemes and results in more uniform relay usage than conventional relay selection scheme. The proposed scheme also results in reduction of relay switching. This further reduces control information to be transmitted for selection and saving energy due to powering up and powering down of the relay and improves network lifetime. We also analyze random relay selection in terms of outage probability, relay usage and relay switching and discuss the trade off between different schemes.

4.2 System Model

Consider a multiple access relay network, shown in Fig. 4.1 , where the source nodes S_1 and S_2 transmit independent message packets $\mathbf{m}_{s_1}$ and $\mathbf{m}_{s_2}$, respectively, to a common destination D through orthogonal channels (in time or frequency) with the help of two relay nodes R_1 and R_2. One relay is selected to assist the sources by forwarding network coded packet $\mathbf{m}_r = \mathbf{m}_{s_1} \oplus \mathbf{m}_{s_2}$, where $\mathbf{m}_r$ denotes network coded packet from selected relay r, $r \in \{R_1, R_2\}$. A relay forwards a network coded packet only if it is able to decode both the sources correctly and is selected for cooperation. If none of the relays are able to decode both the sources successfully, then they

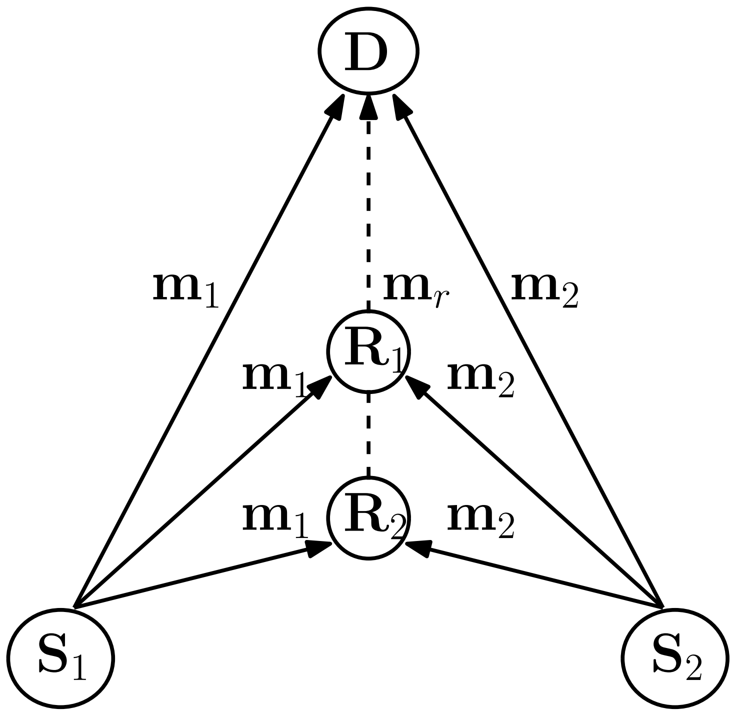

Figure 4.1 System Model for Relay Selection

both remain silent to avoid error propagation. Let

$$\begin{aligned} \mathbf{y}_{s_i d} &= h_{s_i d}\sqrt{E_{s_i}}\ \mathbf{m}_{s_i} + \mathbf{n}_{s_i d}, \quad i \in \{1,2\} \\ \mathbf{y}_{rd} &= h_{rd}\sqrt{E_r}\ \mathbf{m}_r + \mathbf{n}_{rd}, \quad r \in \{R_1, R_2\} \\ \mathbf{y}_{s_i r_j} &= h_{s_i r_j}\sqrt{E_{s_i}}\ \mathbf{m}_{s_i} + \mathbf{n}_{s_i r_j}, \quad i \in \{1,2\}, j \in \{1,2\} \end{aligned} \tag{4.1}$$

denote the received signals at D and R respectively. $h_{s_i d}$ is the channel gain between the source s_i and the destination, modeled as $\mathcal{CN}\left(0, d_{s_i d}^{-\alpha}\right)$, where $\mathcal{CN}(0,1)$ denotes a complex Gaussian distribution with mean 0 and variance 1, α is the path loss exponent and $d_{s_i d}$ is the distance between the source S_i and the destination; $h_{r_j d}$ is the channel gain between the relay R_j and destination, modeled as $\mathcal{CN}\left(0, d_{r_j d}^{-\alpha}\right)$, $d_{r_j d}$ is the distance between the relay R_j and the destination; $h_{s_i r_j}$ is the channel gain between the i^{th} source node and the j^{th} relay, modeled as $\mathcal{CN}\left(0, d_{s_i r_j}^{-\alpha}\right)$ where $d_{s_i r_j}$ is the distance between i^{th} source node and the j^{th} relay. $\mathbf{n}_{s_i d}$,$\mathbf{n}_{r_j d}$ and $\mathbf{n}_{s_i r_j}$ are the AWGN noise vectors, modeled as $\mathcal{CN}(0, N_0)$; E_{s_i} and E_{r_j} are the transmit symbol energies from the source S_i and the relay R_j respectively.

4.3 Relay Selection Scheme

4.3.1 Reliability Based Relay Selection

In this section we will describe the proposed scheme. Consider first the case when both the relays are able to decode both $\mathbf{m}_{s_1}$ and $\mathbf{m}_{s_2}$ are decoded correctly by the relays and the coded packet $\mathbf{m}_r = \mathbf{m}_{s_1} \oplus \mathbf{m}_{s_2}$ is sent to D by the selected relay. Let $m_{s_i}^k, m_r^k$ denote the k^{th} bit of $\mathbf{m}_{s_i}, \mathbf{m}_r$ and $y_{s_i}^k, y_r^k$ denote the k^{th} received bit of $\mathbf{y}_{s_i}, \mathbf{y}_r$ respectively, $i = 1, 2$, $\mathbf{y}_r \in \{\mathbf{y}_{r_1 d}, \mathbf{y}_{r_2 d}\}$. Then the log likelihood ratio (LLR) of m_1^k, m_2^k at D given $\mathbf{h} = (h_{s_1 d}, h_{s_2 d}, h_{rd})$, $h_{rd} \in \{h_{r_1 d}, h_{r_2 d}\}$ and $\mathbf{y}^k = (y_{s_1}^k, y_{s_2}^k, y_r^k)$ is given by [20]

$$\begin{aligned} L(m_{s_1}^k|\mathbf{h}, \mathbf{y}^k) &= L(m_{s_1}^k|h_{s_1 d}, y_{s_1}^k) + L(m_r^k \oplus m_{s_2}^k|h_{s_2 d}, y_{s_2}^k, h_{rd}, y_r^k) \\ L(m_{s_2}^k|\mathbf{h}, \mathbf{y}^k) &= L(m_{s_2}^k|h_{s_2 d}, y_{s_2}^k) + L(m_r^k \oplus m_{s_1}^k|h_{s_1 d}, y_{s_1}^k, h_{rd}, y_r^k) \end{aligned} \tag{4.2}$$

where $L(m_{s_1}^k|h_{s_1 d}, y_{s_1}^k)$ and $L(m_{s_2}^k|h_{s_2 d}, y_{s_2}^k)$ are the LLRs of $m_{s_1}^k$, $m_{s_2}^k$ provided by the direct S_1-D and S_2-D links respectively. $L(m_r^k \oplus m_{s_2}^k|h_{s_2 d}, y_2^k, h_{rd}, y_r^k)$ and $L(m_r^k \oplus m_{s_1}^k|h_{s_1 d}, y_{s_1}^k, h_{rd}, y_r^k)$ is the *additional reliability* provided by the R-D link for $m_{s_1}^k$ and $m_{s_2}^k$ respectively. The second terms of eq.(4.2) can be approximated as [20]

$$\begin{aligned} |L(m_r^k \oplus m_{s_1}^k|h_{s_1 d}, y_{s_1}^k, h_{rd}, y_r^k)| &\approx \min\{|L(m_r^k|h_{rd}, y_r^k)|, |L(m_{s_1}^k|h_{s_1 d}, y_{s_1}^k)|\} \\ |L(m_r^k \oplus m_{s_2}^k|h_{s_2 d}, y_{s_2}^k, h_{rd}, y_r^k)| &\approx \min\{|L(m_r^k|h_{rd}, y_r^k)|, |L(m_{s_2}^k|h_{s_2 d}, y_{s_2}^k)|\} \end{aligned} \tag{4.3}$$

Here

$$L(m_{s_i}^k|h_{s_i d}, y_{s_i}^k) = \frac{4\sqrt{E_{s_i}}}{N_0} \Re e\{h_{s_i d}^* y_{s_i}^k\} \tag{4.4}$$

where $\Re e\{\cdot\}$ denotes the real part of a complex number and $h_{s_i d}^*$ denotes the complex conjugate of $h_{s_i d}$. The magnitude of LLR when averaged over the Gaussian noise is given by

$$E\left[|L(m_{s_i}^k|h_{s_i d}, y_{s_i}^k)|\right] = 4|h_{s_i d}|^2\gamma_{s_i} + 4\sqrt{\frac{\gamma_{s_i}|h_{s_i d}|^2}{\pi}} e^{-\gamma_{s_i}|h_{s_i d}|^2} - 8\gamma_{s_i}|h_{s_i d}|^2 Q\left(\sqrt{2\gamma_{s_i}|h_{s_i d}|^2}\right) \tag{4.5}$$

where $\gamma_{s_i} = E_{s_i}/N_0$ and $Q(x)$ denotes the complementary unit Gaussian distribution function which is equal to $\frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2)\, dt$. Therefore we can approximate $E[|L(m_{s_i}^k|h_{s_i d}, y_{s_i}^k)|] \approx 4|h_{s_i d}|^2\gamma_{s_i}$ and this approximation is fairly accurate even for low γ_{s_i}. Hence the equation (4.3) can be expressed as

$$|L(m_r^k \oplus m_{s_1}^k|h_{s_1d}, y_{s_1}^k, h_{rd}, y_r^k)| \approx \min\{4|h_{s_1d}|^2\gamma_{s_1d}, 4|h_{rd}|^2\gamma_{rd}\}$$
$$|L(m_r^k \oplus m_{s_2}^k|h_{s_2d}, y_{s_2}^k, h_{rd}, y_r^k)| \approx \min\{4|h_{s_2d}|^2\gamma_{s_2d}, 4|h_{rd}|^2\gamma_{rd}\} \quad (4.6)$$

Assuming equal transmission energy from the sources and relays, $E_{s_1} = E_{s_2} = E_{r_1} = E_{r_2}$, we have

$$|L(m_r^k \oplus m_{s_1}^k|h_{s_1d}, y_{s_1}^k, h_{rd}, y_r^k)| \propto \min\{|h_{s_1d}|^2, |h_{rd}|^2\}$$
$$|L(m_r^k \oplus m_{s_2}^k|h_{s_2d}, y_{s_2}^k, h_{rd}, y_r^k)| \propto \min\{|h_{s_2d}|^2, |h_{rd}|^2\} \quad (4.7)$$

Clearly the relay selection rule must be such that the selected relay must provide maximum additional reliability to the sources. However as shown by equation (4.7), this additional reliability is limited by direct source-destination channel gain if the relay-destination link is stronger than source-destination link. This observation motivates us to propose relay selection rule which takes into account the direct source-destination link gains also. We need to ensure that maximum additional reliability can be provided by the selected relay, and hence the selected relay should have stronger link to destination than all the other sources. Therefore we propose following relay selection rule:

$$\text{Choose } r \text{ such that } |h_{rd}|^2 \geq \max\{|h_{s_1d}|^2, |h_{s_2d}|^2\},\ r \in \{R_1, R_2\}. \quad (4.8)$$

If both the relays satisfy above criteria, then any one of them can be chosen at random or the same relay as chosen in previous time slot can be selected in current time slot to avoid unnecessary switching. If none of the relays satisfy above criteria then

$$\text{Choose } r = \arg\max_r |h_{rd}|^2,\ r \in \{R_1, R_2\} \quad (4.9)$$

We choose strongest relay since that will provide maximum additional reliability to both the sources according to Eqs.(4.2) and (4.7). We will call this scheme as *Reliability Based Relay Selection (RbRS)*.

4.3.2 Random Relay Selection

Under this selection rule, when both the relays are able to decode both sources correctly then one of the relay is chosen randomly, irrespective of their channel strengths, for second phase. We analyze this rule also since this rule gives best relay usage asymptotically. We will refer to this scheme as *Random Relay Selection (RRS)*.

4.3.3 Conventional Relay Selection

This selection rule was first proposed by Bletsas et.al [33] for AF protocol and later by Peng et.al[38] for NC. Under this selection rule, when both the relays are able to decode both sources correctly then the relay which has strongest relay-destination channel strength is selected for second phase.

$$\text{Choose } r = \arg\max_{r} |h_{rd}|^2, \; r \in \{R_1, R_2\} \tag{4.10}$$

We will refer to this scheme as *Conventional Relay Selection (CRS)*.

For all the above schemes, if only one relay is able to decode both the sources correctly, then it is selected for second phase. If none of the relays are able to decode both sources correctly, then none of the relays are selected.

4.4 Mathematical Analysis

In this section we will derive the relay usage and relay switching for the proposed scheme. Using simulations and analytical results we show that the proposed scheme provides better relay usage than conventional scheme. Finally we will show that the relay switching is reduced by using the proposed scheme. From simulations we can see that the outage probability for the proposed scheme is same as that of conventional relay selection scheme.

We will denote the channel gains as $g_{s_1d} = |h_{s_1d}|^2$, $g_{s_2d} = |h_{s_2d}|^2$, $g_{r_1d} = |h_{r_1d}|^2$, $g_{r_2d} = |h_{r_2d}|^2$, $g_{s_1r_1} = |h_{s_1r_1}|^2$, $g_{s_2r_1} = |h_{s_2r_1}|^2$, $g_{s_1r_2} = |h_{s_1r_2}|^2$ and $g_{s_2r_2} = |h_{s_2r_2}|^2$. All these channel gains are mutually independent and are exponentially distributed. First consider a point to point communication system as the base line communication system. The received signal at the destination is given by

$$y = hx + n \tag{4.11}$$

The instantaneous signal to noise ratio (SNR) of the channel is given by $|h|^2 E_s/N_0$, $|h|^2$ is exponentially distributed. When the instantaneous SNR is less than certain threshold, the source is said to be in *outage*. The outage probability is given by:

$$P^o(R) = Pr\left(|h|^2 < \frac{2^R - 1}{d^{-\alpha}E_s/N_0}\right) = 1 - e^{\left(-\frac{2^R-1}{d^{-\alpha}E_s/N_0}\right)} \tag{4.12}$$

where R is the spectral efficiency of the system in bits per channel use.

4.4.1 Relay Usage

In this section we will discuss about relay usage. In a power constrained network, balanced (approximately equal) usage of relay nodes is important since it maximizes network lifetime and connectivity. An unbalanced relay usage leads to lost of connectivity since some relay nodes run out of battery and leads to higher outage probability. The relay usage for the proposed scheme can be calculated by summing the probabilities when respective relays are selected.

Let $P_{success}$ denotes the probability of both the sources being successfully decoded at both the relays

$$P_{success} = P^{oc}_{s_1r_1}P^{oc}_{s_2r_1}P^{oc}_{s_1r_2}P^{oc}_{s_2r_2} \tag{4.13}$$

Let $\lambda_{s_1d} \triangleq d^{\alpha}_{s_1d}$, $\lambda_{s_2d} \triangleq d^{\alpha}_{s_2d}$, $\lambda_{r_1d} \triangleq d^{\alpha}_{r_1d}$ and $\lambda_{r_2d} \triangleq d^{\alpha}_{r_2d}$. We assume that each source transmits information at the spectral efficiency of R bits per channel use. We have following disjoint events:

1. $\mathcal{E}_1$: None of the relays are able to decode both the sources successfully.
2. $\mathcal{E}_2$: Only R_1 decodes both sources successfully.
3. $\mathcal{E}_3$: Only R_2 decodes both sources successfully.
4. $\mathcal{E}_4$: Both R_1 and R_2 decode both the sources successfully. Both R_1-D and R_2-D channel gains are greater than S_1-D and S_2-D channel gains and R_1 is selected randomly.
5. $\mathcal{E}_5$: Both R_1 and R_2 decode both the sources successfully. Both R_1-D and R_2-D channel gains are greater than S_1-D and S_2-D channel gains and R_2 is selected randomly.
6. $\mathcal{E}_6$: Both R_1 and R_2 decode both the sources successfully. R_2-D channel gain is less than either S_1-D or S_2-D channel gain or both, and R_1-D channel gain is greater than R_2-D channel gain.
7. $\mathcal{E}_7$: Both R_1 and R_2 decode both the sources successfully. R_1-D channel gain is less than either S_1-D or S_2-D channel gain or both, and R_2-D channel gain is greater than

R_1-D channel gain.

Using above disjoint events, the relay usage of R_1 for RbRS can be calculated as follows:

$$\begin{aligned}
\mathcal{U}_{R_1,RbRS} =& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+ P\left(\min\{g_{r_1},g_{r_2}\} > \max\{g_1,g_2\}\right) \times P_{success} \times \frac{1}{2} \\
&+ P(g_{r_2} < \max\{g_1,g_2\}, g_{r_1} > g_{r_2}) \times P_{success} \\
=& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+ \frac{1}{2} \times P_{success} \times \left(\frac{\lambda_{s_1d}+2\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}} + \frac{\lambda_{s_2d}+2\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_2d}} \right. \\
&\left. - \frac{\lambda_{s_1d}+\lambda_{s_2d}+2\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}+\lambda_{s_2d}}\right)
\end{aligned} \tag{4.14}$$

Similarly relay usage for R_2 is given by

$$\begin{aligned}
\mathcal{U}_{R_2,RbRS} =& \left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\left(1-(1-P^o_{s_1r_1})(1-P^o_{s_2r_1})\right) \\
&+ P\left(\min\{g_{r_1},g_{r_2}\} > \max\{g_1,g_2\}\right) \times P_{success} \times \frac{1}{2} \\
&+ P(g_{r_1} < \max\{g_1,g_2\}, g_{r_2} > g_{r_1}) \times P_{success} \\
=& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+ \frac{1}{2} \times P_{success} \times \left(\frac{\lambda_{s_1d}+2\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}} + \frac{\lambda_{s_2d}+2\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_2d}} \right. \\
&\left. - \frac{\lambda_{s_1d}+\lambda_{s_2d}+2\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}+\lambda_{s_2d}}\right)
\end{aligned} \tag{4.15}$$

Now we will calculate relay usage for conventional relay selection scheme. Since the conventional relay scheme selects the relay which has strongest R-D channel gain, the relay usage for R_1 and R_2 for conventional relay selection scheme can be calculated as

$$\begin{aligned}
\mathcal{U}_{R_1,CRS} =& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) + P\left(g_{r_1} > g_{r_2}\right) \times P_{success} \\
=& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) + P_{success}\,\frac{\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}}
\end{aligned} \tag{4.16}$$

where $\mathcal{U}_{R_1}$ denotes the probability of selecting R_1. Similarly relay usage for R_2 is given by

$$\begin{aligned}
\mathcal{U}_{R_2,CRS} =& \left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\left(1-(1-P^o_{s_1r_1})(1-P^o_{s_2r_1})\right) + P\left(g_{r_1} < g_{r_2}\right) \times P_{success} \\
=& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) + P_{success}\,\frac{\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}}
\end{aligned} \tag{4.17}$$

Now we will calculate relay usage for random relay selection scheme. Since the random relay scheme selects the relay randomly, the relay usage for R_1 and R_2 for the scheme can be calculated as

$$\mathcal{U}_{R_1,RRS}= \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P_{success} \times \frac{1}{2} \quad (4.18)$$

$$\mathcal{U}_{R_2,RRS}= \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P_{success} \times \frac{1}{2} \quad (4.19)$$

As shown by simulation results, the theoretical analysis matches with simulations and the proposed scheme has much more balanced relay usage than the conventional scheme. Overall the random relay selection provides the most balanced relay usage amongst all the three schemes.

4.4.2 Relay Switching Rate

In this section we will discuss about relay switching rate. In a cooperative network with relay selection, the selected relay might be different from one time instant to next. The leads to powering down of other non-selected relays. The powering up and down of relay nodes leads to wastage of energy. Also lot of control information might be needed to send if the relays are switched frequently. Hence its important to reduce relay switching as much as possible.

We will first analyze Relay Switching for RbRS. We will use Markov model to analyze switching rate. Let a state space be denoted as $\mathcal{S}$ and the state at time instant t be denoted by $\mathcal{S}_t$. The possible states are $\{\mathcal{S}\} := \{1, 2, 3\}$, $\mathcal{S}_t = 1$ when R_1 is selected, $\mathcal{S}_t = 2$ when R_2 is selected and $\mathcal{S}_t = 3$ when none of the relays is selected. Suppose at time instant t, $\mathcal{S}_t = 1$. For time instant $t+1$, one of the following three possibilities can happen for the proposed selection rule:

1. The state remains the same ie $\mathcal{S}_{t+1} = 1$ ie R_1 is selected again for the transmission. This can occur if any of the following three events occur:
   (a) Only R_1 decodes both sources successfully.
   (b) Both R_1 and R_2 decode both the sources successfully and both R_1-D and R_2-D channel gains are greater than S_1-D and S_2-D channel gains. Hence again R_1 is selected to avoid unnecessary switching.

   (c) Both R_1 and R_2 decode both the sources successfully and R_2-D channel gain is less than either S_1-D or S_2-D channel gain or both, and R_1-D channel gain is greater than R_2-D channel gain.

2. The state changes to $\mathcal{S}_{t+1} = 2$ ie R_2 is selected again for the transmission. This can occur if any of the following two events occur:

   (a) Only R_2 decodes both sources successfully.

   (b) Both R_1 and R_2 decode both the sources successfully and R_1-D channel gain is less than either S_1-D or S_2-D channel gain or both, and R_2-D channel gain is greater than R_1-D channel gain.

3. The state changes to $\mathcal{S}_{t+1} = 3$ ie none of the relays are selected since none of them were able to decode both the sources successfully.

Similarly we can define transitions from states $\mathcal{S}_t = 2$, $\mathcal{S}_t = 3$ and a transition probability matrix $\mathcal{T} \triangleq [f_{ij}]$. The elements of transition probability matrix are the conditional transition probabilities defined as $f_{ij} = P(\mathcal{S}_{t+1} = j|\mathcal{S}_t = i)$, $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}$. The Markov states are shown in Fig.4.2.

Now we will calculate the individual transition probabilities.

$$\begin{aligned}
f_{11} =& \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) \\
&+ P\left(\min\{g_{r_1}, g_{r_2}\} > \max\{g_1, g_2\}\right) \times P_{success} \\
&+ P(g_{r_2} < \max\{g_1, g_2\}, g_{r_1} > g_{r_2}) \times P_{success} \\
=& \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) \\
&+ P_{success} \times \left(\frac{\lambda_{s_1d} + \lambda_{r_2d}}{\lambda_{r_1d} + \lambda_{r_2d} + \lambda_{s_1d}} + \frac{\lambda_{s_2d} + \lambda_{r_2d}}{\lambda_{r_1d} + \lambda_{r_2d} + \lambda_{s_2d}} - \frac{\lambda_{s_1d} + \lambda_{s_2d} + \lambda_{r_2d}}{\lambda_{r_1d} + \lambda_{r_2d} + \lambda_{s_1d} + \lambda_{s_2d}}\right)
\end{aligned} \tag{4.20}$$

$$\begin{aligned}
f_{12} =& \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) \\
&+ P(g_{r_1} < \max\{g_1, g_2\}, g_{r_2} > g_{r_1}) \times P_{success} \\
=& \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) \\
&+ P_{success} \times \left(\frac{\lambda_{r_1d}}{\lambda_{r_1d} + \lambda_{r_2d} + \lambda_{s_1d}} + \frac{\lambda_{r_1d}}{\lambda_{r_1d} + \lambda_{r_2d} + \lambda_{s_2d}} - \frac{\lambda_{r_1d}}{\lambda_{r_1d} + \lambda_{r_2d} + \lambda_{s_1d} + \lambda_{s_2d}}\right)
\end{aligned} \tag{4.21}$$

$$f_{13} = \left(1 - \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\right)\left(1 - \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\right) \tag{4.22}$$

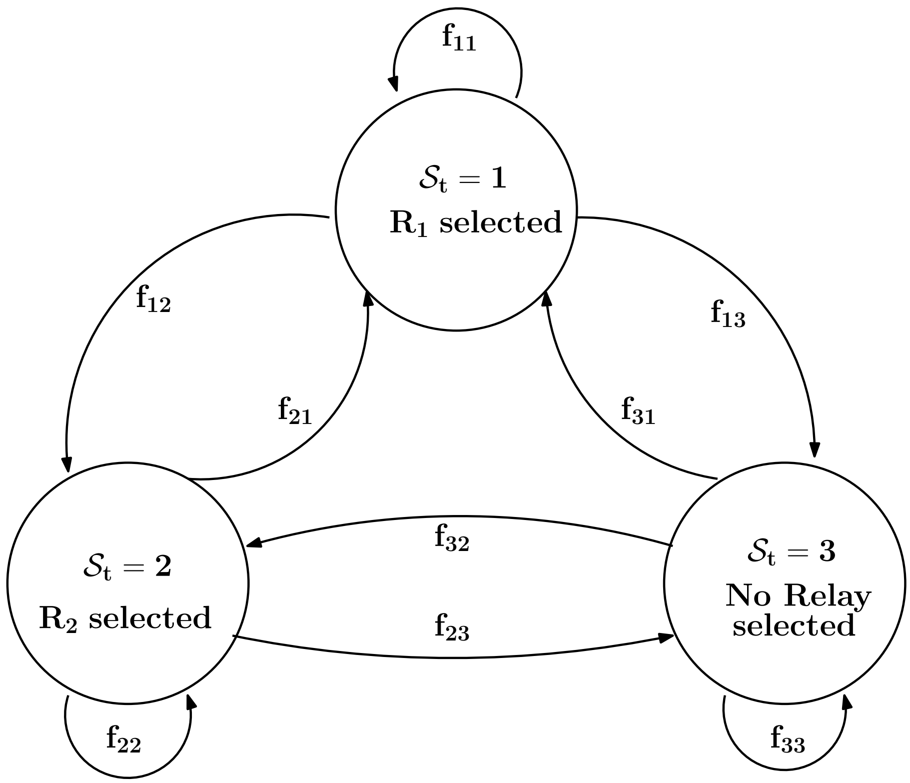

Figure 4.2 Markov states for relay switching with RbRS scheme.

$$
\begin{aligned}
f_{21} =& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+P(g_{r_2}<\max\{g_1,g_2\}, g_{r_1}>g_{r_2})\times P_{success} \\
=& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+P_{success}\times\left(\frac{\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}}+\frac{\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_2d}}-\frac{\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}+\lambda_{s_2d}}\right)
\end{aligned} \tag{4.23}
$$

$$
\begin{aligned}
f_{22} =& \left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\left(1-(1-P^o_{s_1r_1})(1-P^o_{s_2r_1})\right) \\
&+P\left(\min\{g_{r_1},g_{r_2}\}>\max\{g_1,g_2\}\right)\times P_{success} \\
&+P(g_{r_1}<\max\{g_1,g_2\}, g_{r_2}>g_{r_1})\times P_{success} \\
=& \left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\left(1-(1-P^o_{s_1r_1})(1-P^o_{s_2r_1})\right) \\
&+P_{success}\times\left(\frac{\lambda_{s_1d}+\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}}+\frac{\lambda_{s_2d}+\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_2d}}-\frac{\lambda_{s_1d}+\lambda_{s_2d}+\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}+\lambda_{s_2d}}\right)
\end{aligned} \tag{4.24}
$$

$$
f_{23} = \left(1-\left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\right)\left(1-\left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\right) \tag{4.25}
$$

$$
\begin{aligned}
f_{31} =& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+P(g_{r_2}<\max\{g_1,g_2\}, g_{r_1}>g_{r_2})\times P_{success} \\
&+P\left(\min\{g_{r_1},g_{r_2}\}>\max\{g_1,g_2\}\right)\times P_{success}\times\frac{1}{2} \\
=& \left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\left(1-(1-P^o_{s_1r_2})(1-P^o_{s_2r_2})\right) \\
&+\frac{1}{2}\times P_{success}\times\left(\frac{\lambda_{s_1d}+2\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}}+\frac{\lambda_{s_2d}+2\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_2d}}-\frac{\lambda_{s_1d}+\lambda_{s_2d}+2\lambda_{r_2d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}+\lambda_{s_2d}}\right)
\end{aligned} \tag{4.26}
$$

$$
\begin{aligned}
f_{32} =& \left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\left(1-(1-P^o_{s_1r_1})(1-P^o_{s_2r_1})\right) \\
&+P(g_{r_1}<\max\{g_1,g_2\}, g_{r_2}>g_{r_1})\times P_{success} \\
&+P\left(\min\{g_{r_1},g_{r_2}\}>\max\{g_1,g_2\}\right)\times P_{success}\times\frac{1}{2} \\
=& \left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\left(1-(1-P^o_{s_1r_1})(1-P^o_{s_2r_1})\right) \\
&+\frac{1}{2}\times P_{success}\times\left(\frac{\lambda_{s_1d}+2\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}}+\frac{\lambda_{s_2d}+2\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_2d}}-\frac{\lambda_{s_1d}+\lambda_{s_2d}+2\lambda_{r_1d}}{\lambda_{r_1d}+\lambda_{r_2d}+\lambda_{s_1d}+\lambda_{s_2d}}\right)
\end{aligned} \tag{4.27}
$$

$$
f_{33} = \left(1-\left(1-P^o_{s_1r_1}\right)\left(1-P^o_{s_2r_1}\right)\right)\left(1-\left(1-P^o_{s_1r_2}\right)\left(1-P^o_{s_2r_2}\right)\right) \tag{4.28}
$$

Notice that for a given SNR, all the transition probabilities are finite, non zero and independent of number of transitions. Hence all the states are persistent and aperiodic and therefore the associated Markov chain has a unique stationary distribution. Suppose that the stationary distribution for the proposed relay selection schemes is denoted as $\vec{\pi}$ and π_i denote the stationary probability of the i^{th} state. The stationary probabilities can be obtained in closed form by solving following simultaneous linear equations:

$$\begin{cases} \vec{\pi}\mathcal{T} = \vec{\pi} \\ \sum_{i=1}^{3} \pi_i = 1 \end{cases} \tag{4.29}$$

Solving above equations, the stationary distribution of the proposed scheme can be obtained as

$$\begin{aligned} \pi_1 &= \frac{f_{13}f_{31} + f_{21} - f_{13}f_{21}}{f_{12} + f_{13} + f_{21}} \\ \pi_2 &= \frac{(1 - f_{13})(f_{12} + f_{13}) - f_{13}f_{31}}{f_{12} + f_{13} + f_{21}} \\ \pi_3 &= f_{13} \end{aligned} \tag{4.30}$$

The overall relay switching rate for proposed relay selection scheme can be calculated as

$$\begin{aligned} P_{S,RbRS} &= P(\mathcal{S}_{t+1} = 2|\mathcal{S}_t = 1)P(\mathcal{S}_t = 1) + P(\mathcal{S}_{t+1} = 1|\mathcal{S}_t = 2)P(\mathcal{S}_t = 2) \\ &= f_{12}P(\mathcal{S}_t = 1) + f_{21}P(\mathcal{S}_t = 2) \\ &= f_{12}\pi_1 + f_{21}\pi_2 \end{aligned} \tag{4.31}$$

4.4.2.1 Relay Switching for Conventional Selection Rule

Let $\mathcal{R}_t$ denotes the relay selected at time instant t. Suppose at time instant t, $\mathcal{R}_t = R_1$. For time instant $t + 1$, one of the following three possibilities can happen for the conventional selection rule:

1. R_1 is selected again for the transmission. This can occur if any of the following two events occur:
   (a) Only R_1 decodes both sources successfully.
   (b) Both R_1 and R_2 decode both the sources successfully and R_1-D channel gains is greater than R_2-D channel gain.
2. R_2 is selected for the transmission. This can occur if any of the following two events occur:

(a) Only R_2 decodes both sources successfully.

(b) Both R_1 and R_2 decode both the sources successfully and R_2-D channel gains is greater than R_1-D channel gain.

3. None of the relays are selected since none of them were able to decode both the sources successfully.

Similarly we can define switching from $\mathcal{R}_t = R_2$. Let p_{ij} denote the conditional probability that given $\mathcal{R}_t = R_i$, $\mathcal{R}_{t+1} = R_j$ where $i, j \in \{1, 2\}$.

$$\begin{aligned} p_{11} &= \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P\left(g_{r_1} > g_{r_2}\right) \times P_{success} \\ &= \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P_{success} \times \frac{\lambda_{r_2d}}{\lambda_{r_1d} + \lambda_{r_2d}} \end{aligned} \quad (4.32)$$

$$\begin{aligned} p_{12} &= \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P\left(g_{r_2} > g_{r_1}\right) \times P_{success} \\ &= \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P_{success} \times \frac{\lambda_{r_1d}}{\lambda_{r_1d} + \lambda_{r_2d}} \end{aligned} \quad (4.33)$$

$$\begin{aligned} p_{21} &= \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P\left(g_{r_1} > g_{r_2}\right) \times P_{success} \\ &= \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P_{success} \times \frac{\lambda_{r_2d}}{\lambda_{r_1d} + \lambda_{r_2d}} \end{aligned} \quad (4.34)$$

$$\begin{aligned} p_{22} &= \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P\left(g_{r_2} > g_{r_1}\right) \times P_{success} \\ &= \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P_{success} \times \frac{\lambda_{r_1d}}{\lambda_{r_1d} + \lambda_{r_2d}} \end{aligned} \quad (4.35)$$

The overall relay switching rate for conventional relay selection scheme can be calculated as

$$\begin{aligned} P_{S,CRS} &= P(\mathcal{R}_{t+1} = R_2 | \mathcal{R}_t = R_1) P(\mathcal{R}_t = R_1) + P(\mathcal{R}_{t+1} = R_1 | \mathcal{R}_t = R_2) P(\mathcal{R}_t = R_2) \\ &= p_{12} P(\mathcal{R}_t = R_1) + p_{21} P(\mathcal{R}_t = R_2) \\ &= p_{12}\, \mathcal{U}_{R_1,CRS} + p_{21}\, \mathcal{U}_{R_2,CRS} \end{aligned} \quad (4.36)$$

4.4.2.2 Relay Switching for Random Relay Selection Rule

We will analyze the switching for RRS in same way as we did for CRS. Let $\mathcal{R}_t$ denotes the relay selected at time instant t. Suppose at time instant t, $\mathcal{R}_t = R_1$. For time instant $t + 1$, one of the following three possibilities can happen for the conventional selection rule:

1. R_1 is selected again for the transmission. This can occur if any of the following two events occur:
   (a) Only R_1 decodes both sources successfully.
   (b) Both R_1 and R_2 decode both the sources successfully and R_1 is selected randomly.
2. R_2 is selected for the transmission. This can occur if any of the following two events occur:
   (a) Only R_2 decodes both sources successfully.
   (b) Both R_1 and R_2 decode both the sources successfully and R_2 is selected randomly.
3. None of the relays are selected since none of them were able to decode both the sources successfully.

Similarly we can define switching from $\mathcal{R}_t = R_2$. Let q_{ij} denote the conditional probability that given $\mathcal{R}_t = R_i$, $\mathcal{R}_{t+1} = R_j$ where $i, j \in \{1, 2\}$.

$$q_{11} = \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P_{success} \times \frac{1}{2} \tag{4.37}$$

$$q_{12} = \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P_{success} \times \frac{1}{2} \tag{4.38}$$

$$q_{21} = \left(1 - P^o_{s_1r_1}\right)\left(1 - P^o_{s_2r_1}\right)\left(1 - (1 - P^o_{s_1r_2})(1 - P^o_{s_2r_2})\right) + P_{success} \times \frac{1}{2} \tag{4.39}$$

$$q_{22} = \left(1 - P^o_{s_1r_2}\right)\left(1 - P^o_{s_2r_2}\right)\left(1 - (1 - P^o_{s_1r_1})(1 - P^o_{s_2r_1})\right) + P_{success} \times \frac{1}{2} \tag{4.40}$$

The overall relay switching rate for random relay selection scheme can be calculated as

$$\begin{aligned} P_{S,RRS} &= P(\mathcal{R}_{t+1} = R_2 | \mathcal{R}_t = R_1) P(\mathcal{R}_t = R_1) + P(\mathcal{R}_{t+1} = R_1 | \mathcal{R}_t = R_2) P(\mathcal{R}_t = R_2) \\ &= q_{12} P(\mathcal{R}_t = R_1) + q_{21} P(\mathcal{R}_t = R_2) \\ &= q_{12}\, \mathcal{U}_{R_1,RRS} + q_{21}\, \mathcal{U}_{R_2,RRS} \end{aligned} \tag{4.41}$$

4.5 Simulation Results

In this section we present simulation results. We assume that the two sources S_1 and S_2 both are at a distance of 1 from the destination D. The path loss exponent is assumed to be $\alpha = 4$. The transmit energies of both the sources and relay are assumed to be the same, i.e. $E_1 = E_2 = E_r$ and the spectral efficiency is assumed to be 1 bit per channel use(bpcu).

Fig. 4.3 and 4.4 compares the simulation results of outage probabilities of proposed Reliability based Relay Selection (RbRS), Conventional Relay Selection (CRS) and Random Relay Selection (RRS). We assume that the two sources S_1 and S_2 both are at a distance of 1 from the destination D. The path loss exponent is assumed to be $\alpha = 4$. The transmit energies of both the sources and relay are assumed to be the same, i.e. $E_1 = E_2 = E_r$ and the spectral efficiency is assumed to be 1 bit per channel use. In one case, the relay nodes R_1 and R_2 are at distance of 0.8 and 0.9 from destination and in second the relay nodes are at distance of 0.3 and 0.4 from the destination. As shown by simulation results, RbRS provides same outage probability as CRS. We can see that when the relays are closer to destination, then RRS provides almost same outage as other two schemes. However when relays are closer to sources, then its outage probability is higher than the other two schemes.

Fig. 4.5 compares the simulated relay usage of proposed Reliability based Relay Selection (RbRS), Conventional Relay Selection (CRS) and Random Relay Selection (RRS). We can see that RbRS provides much better relay usage than CRS and hence improves network lifetime. Asymptotically RRS provides the best relay usage, each relay being used 50% of the time.

Fig. 4.6 shows the simulated and analytically calculated usage of R_1. The usage for Conventional Relay Selection(CRS) is given by Eq.(4.16). The usage for proposed Reliability based Relay Selection (RbRS) is given by Eq.(4.14). The usage for Random Relay Selection(RRS) is given by Eq.(4.18).

Fig. 4.7 shows the simulated and analytically calculated usage of R_2. The usage for Conventional Relay Selection(CRS) is given by Eq.(4.17). The usage for proposed Reliability based Relay Selection (RbRS) is given by Eq.(4.15). The usage for Random Relay Selection(RRS) is given by Eq.(4.19).

Fig. 4.8 compares the relay switching probability of proposed Reliability based Relay Selection (RbRS), Conventional Relay Selection (CRS) and Random Relay Selection (RRS). We can see that RbRS reduces the switching by half compared to CRS and hence further improves network lifetime. RRS has the highest switching probability amongst all the schemes. We can further see that the analytical results match quite well with simulation results except for low SNR case for RRS. The switching rate for CRS is given by Eq.(4.36), the switching rate for

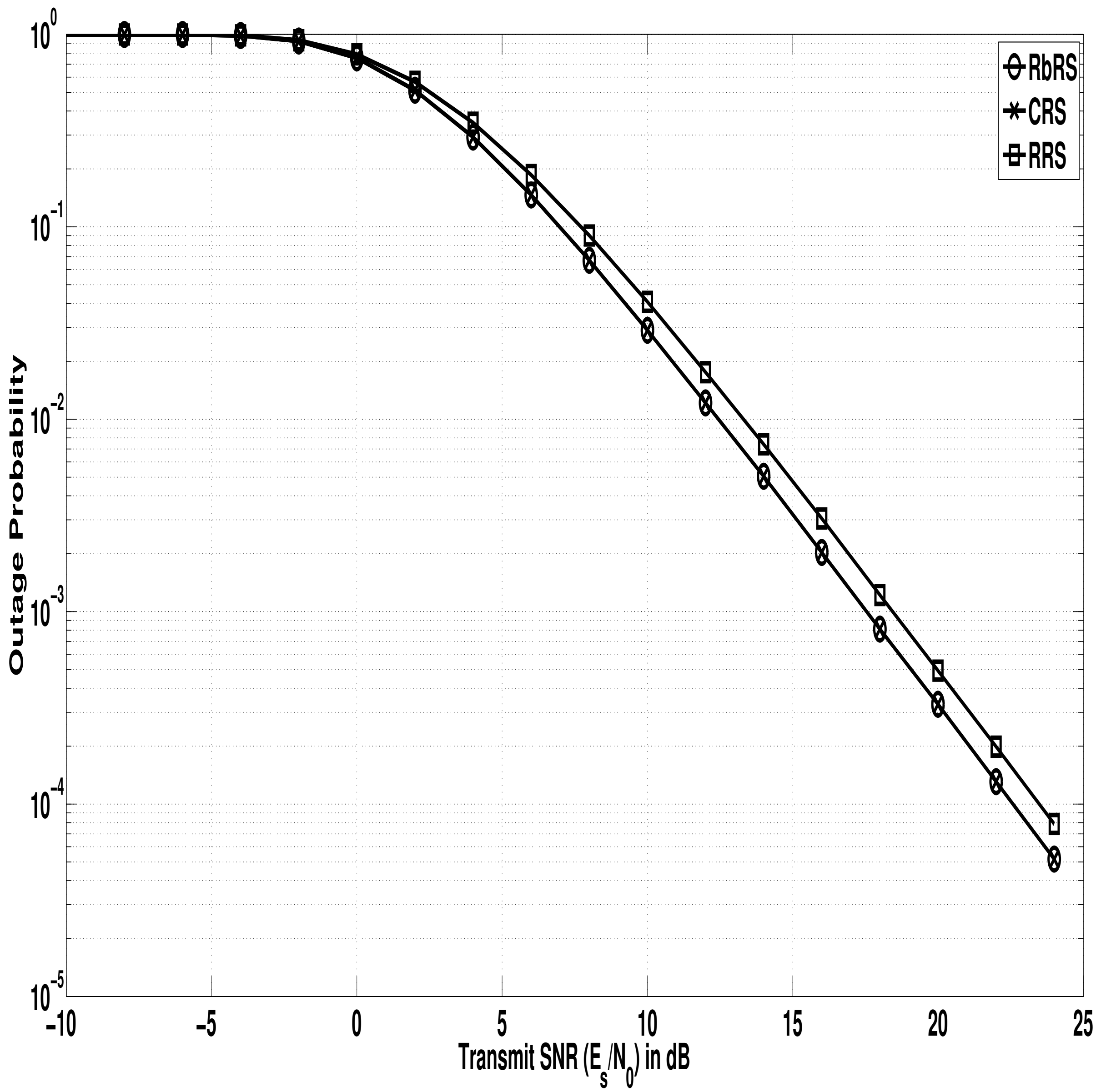

Figure 4.3 Outage Probability comparison of RbRS, CRS and RRS, Spectral Efficiency = 1 bpcu, $d_{r_1d} = 0.8$, $d_{r_2d} = 0.9$.

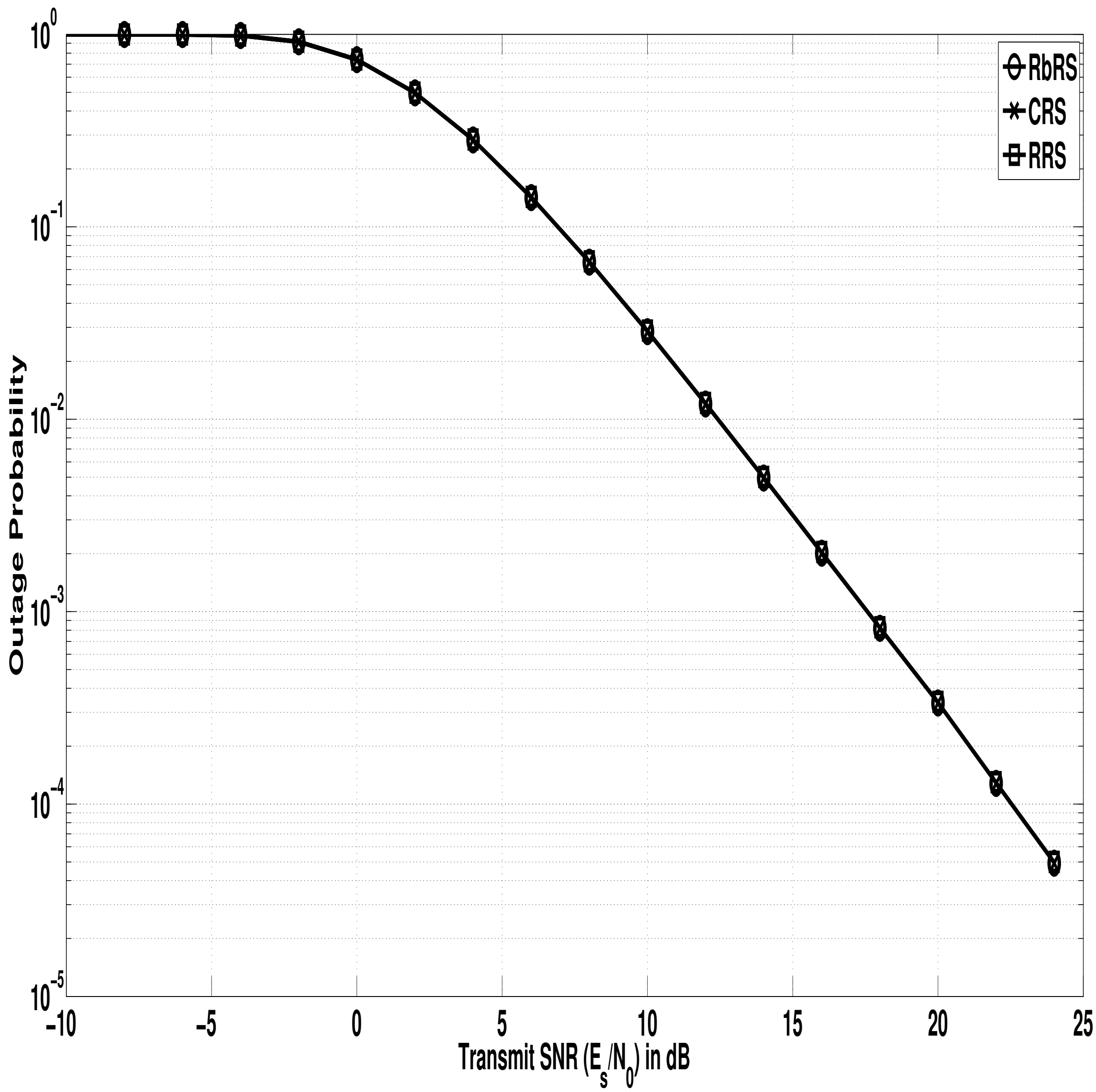

Figure 4.4 Outage Probability comparison of RbRS, CRS and RRS, Spectral Efficiency = 1 bpcu, $d_{r_1 d} = 0.3$, $d_{r_2 d} = 0.4$.

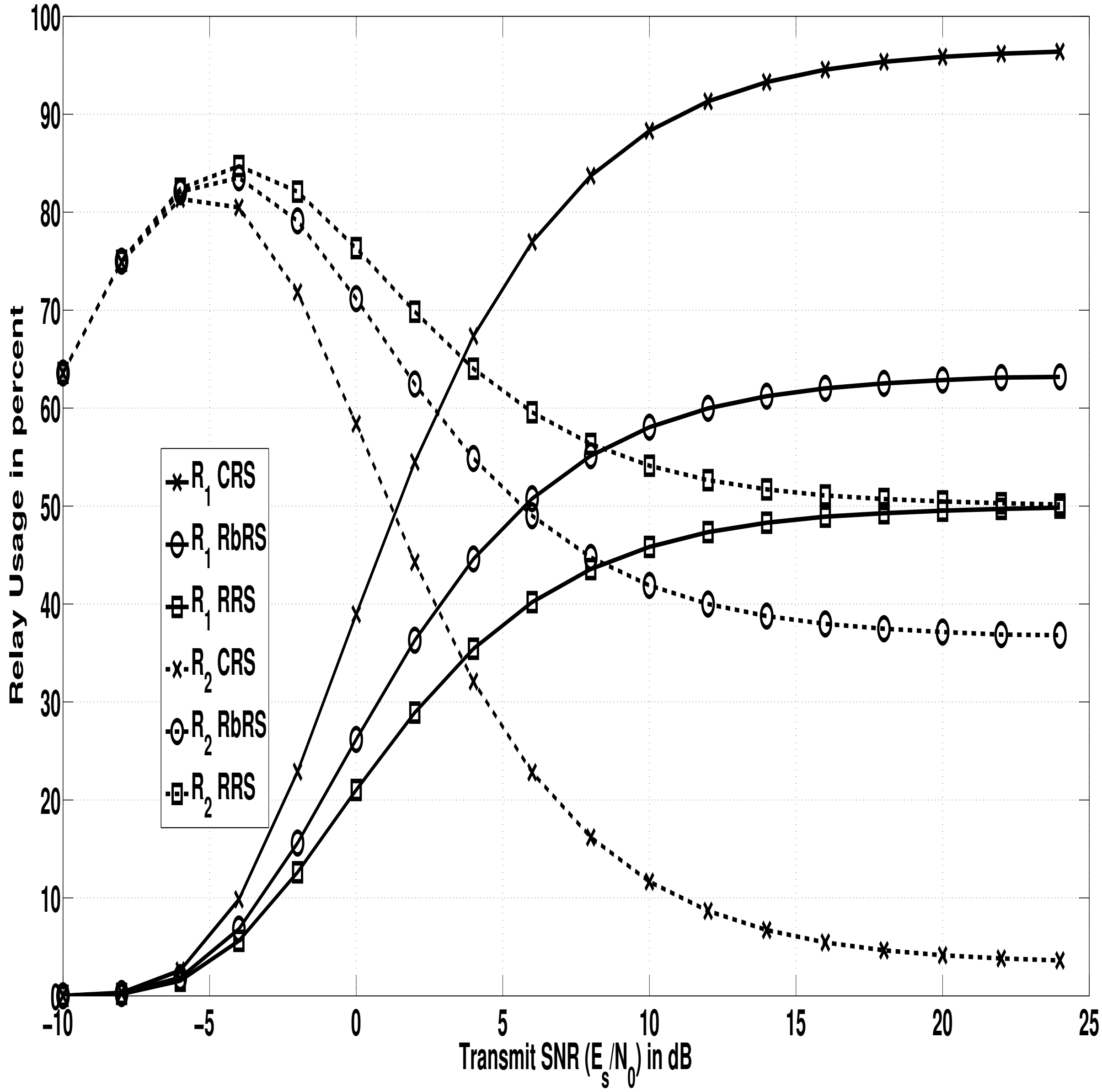

Figure 4.5 Simulated relay usage comparison of RbRS, CRS and RRS, Spectral Efficiency = 1 bpcu, $d_{r_1d} = 0.3$, $d_{r_2d} = 0.7$.

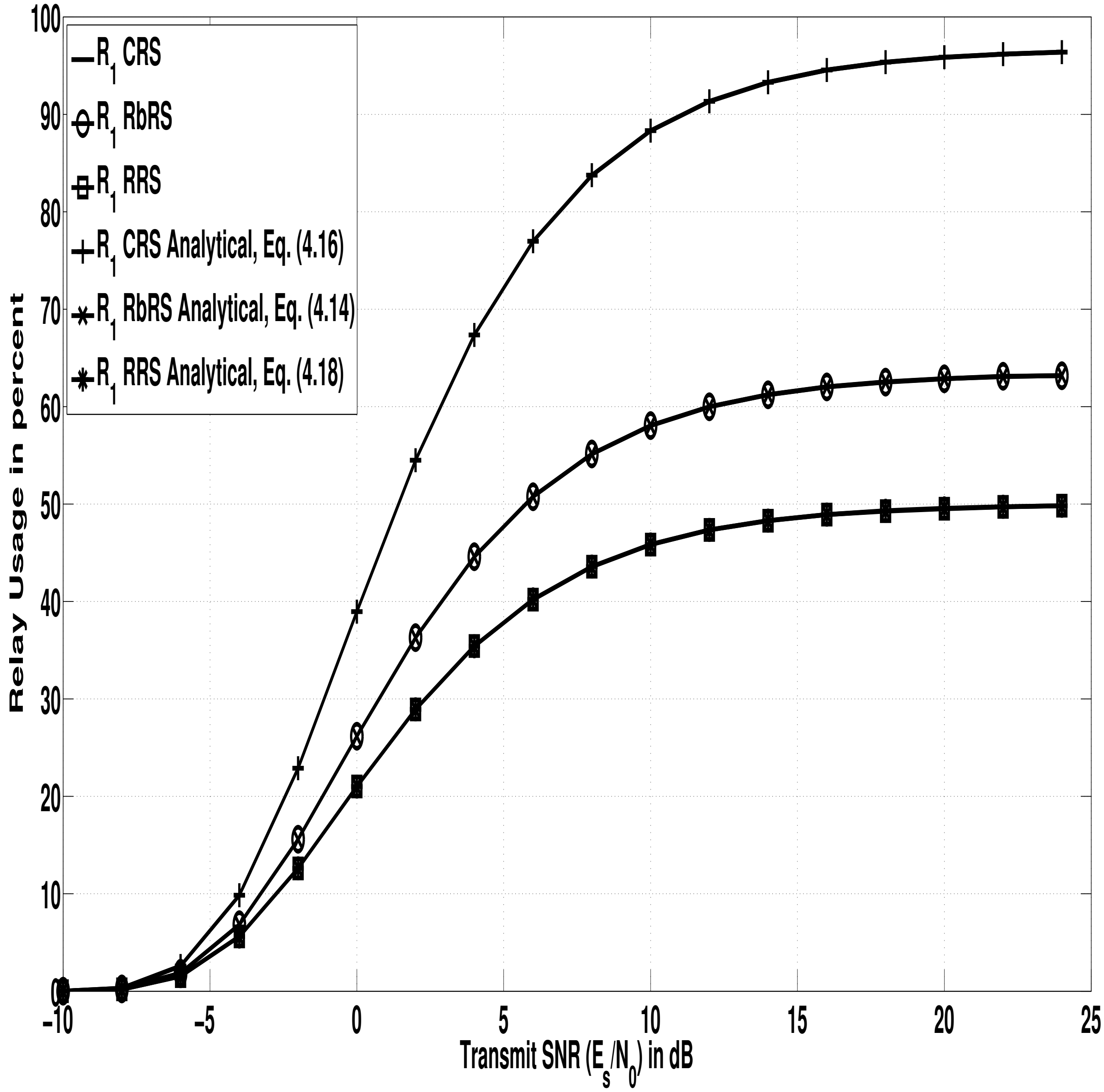

Figure 4.6 Simulated and Analytical relay Usage comparison of R_1 for RbRS, CRS and RRS, Spectral Efficiency = 1 bpcu, $d_{r_1d} = 0.3$, $d_{r_2d} = 0.7$.

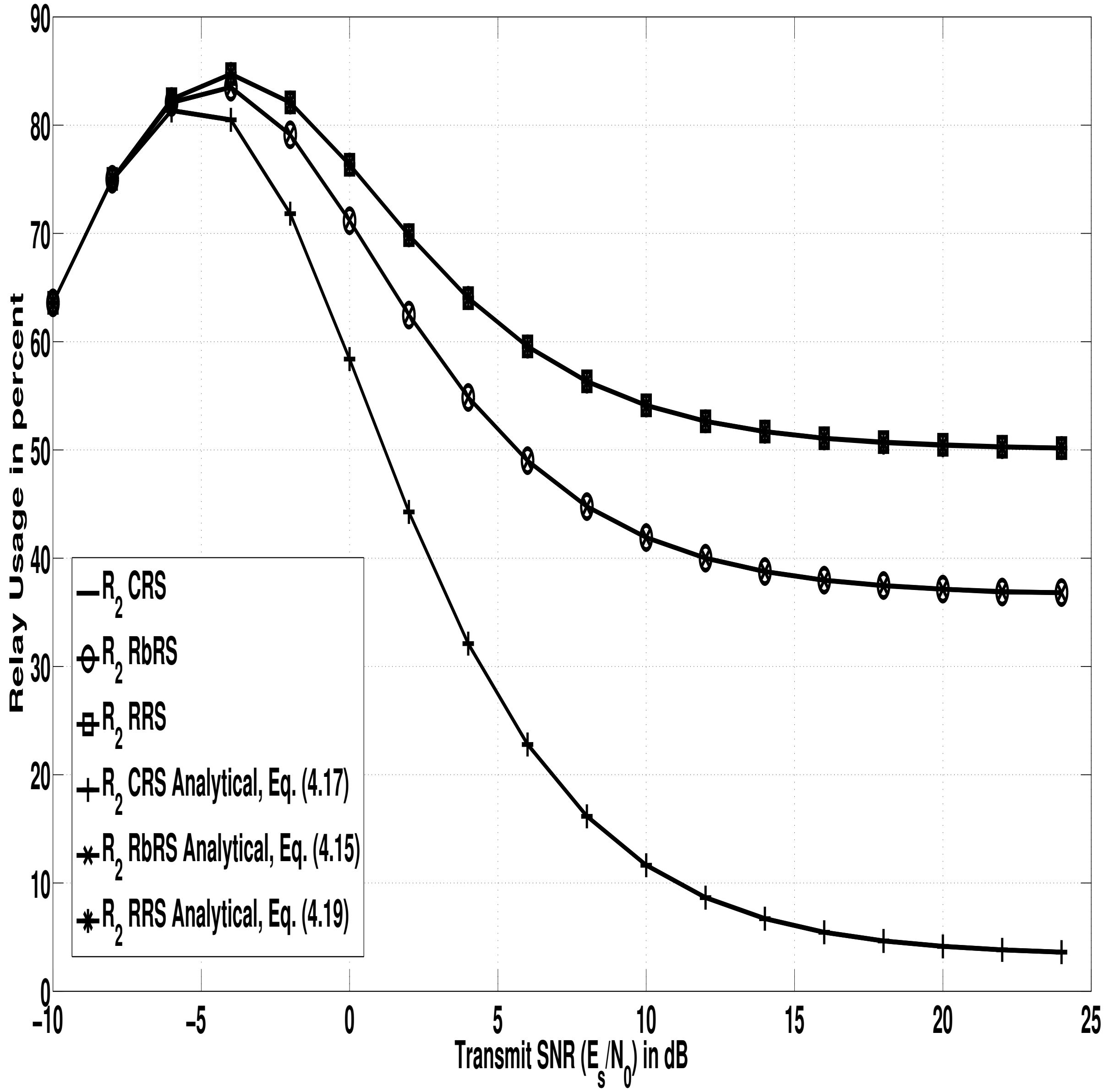

Figure 4.7 Simulated and Analytical relay Usage comparison of R_2 for RbRS, CRS and RRS, Spectral Efficiency = 1 bpcu, $d_{r_1d} = 0.3$, $d_{r_2d} = 0.7$.

RbRS is given by Eq.(4.31) and the switching rate for RRS is given by Eq.(4.41).

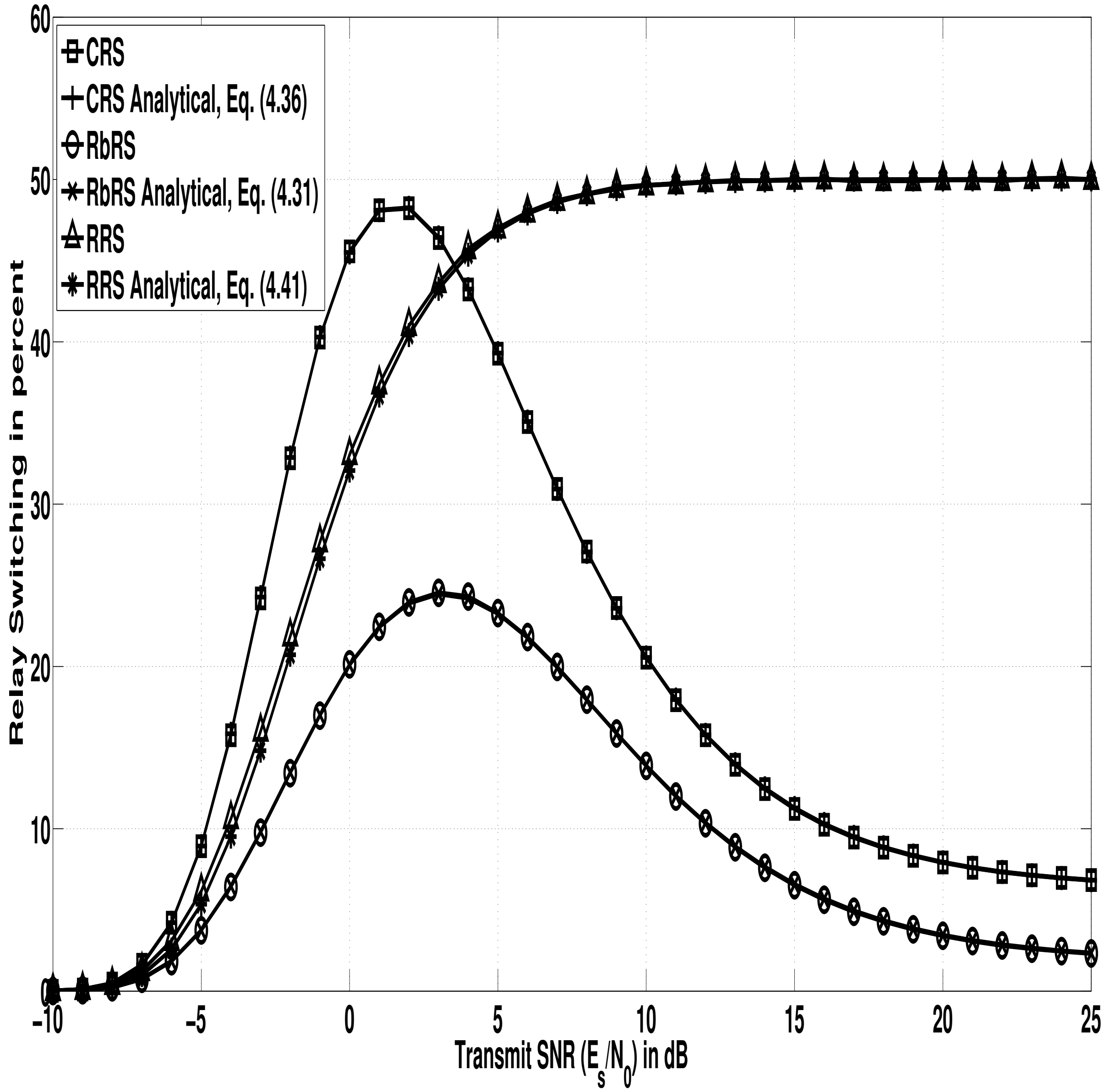

Figure 4.8 Relay Switching comparison of RbRS, CRS and RRS, Spectral Efficiency = 1 bpcu, $d_{r_1 d} = 0.3$, $d_{r_2 d} = 0.7$.

4.6 Conclusion

We have proposed a new relay selection scheme for cooperative network coding scenario which gives exact outage performance as conventional scheme and improves network usage and reduces relay switching in comparison to conventional selection scheme. We also analyzed random relay selection which gives similar outage performance when the relays are closer to destination and has better relay usage, however has very high relay switching rate.

CHAPTER 5. CONCLUSION AND FUTURE WORK

5.1 Conclusion

In this work we investigated three different problems related to cooperative communication and network coding. In the first problem we proposed flexible network coding rules to provide variable QoS to different users of the network. The QoS of the users can be modified dynamically by controlling their individual levels of prioritization. These rules ensure that the maximum diversity order is provided to all the users in the system resulting in improvement in overall throughput compared to existing UEP coding schemes proposed in the literature. We then briefly described how to jointly optimize the prioritization level and location of the relays to provide required QoS to prioritized nodes and provide non-prioritized nodes simultaneously with best possible QoS.

In our second problem, we investigated problem of false injection by malicious relay nodes in cooperative network coding and proposed novel mechanism to mitigate the false injection based on different attack models. The side information of homomorphic hash functions is utilized at the physical layer to combat false injection and improve network throughput by recycling the polluted packets iteratively.

In the final problem we presented a novel relay selection algorithm with takes into account the channel strengths of direct source-destination links for choosing relay for second phase transmission. As shown by theoretical and simulation results, the proposed algorithm provides much uniform relay utilization and improves relay switching without sacrificing outage probability. This results in overall improvement of network lifetime.

5.2 Future Work

The future work for prioritized network coding can be

- Evaluate the performance of proposed soft prioritized network coding rules jointly with practical capacity achieving channel codes such as LDPC codes.
- Extend the problem to multiple destination with different prioritized users/groups.

The future work for recycling scheme can be

- Extend the work to capacity achieving LDPC codes.
- Evaluate the performance with noisy source-relay and relay-destination channel.

The future work for relay selection can be

- Extend the work for multi-source, multi-relay scenario with correlated fading channel model.
- It would be interesting to investigate the proposed scheme in case when the relays use power control.
- Evaluate the network lifetime in combination with ARQ schemes.

APPENDIX A. PROOF OF EQ.(2.8)

In this appendix we will derive Eq.(2.8). Suppose a point to point communication link

$$y = hx + n \tag{A.1}$$

where $x = \{-\sqrt{E_s}, +\sqrt{E_s}\}$ is the transmitted symbol, h is channel gain, n is AWGN noise and y is the received signal. Both h and n are complex Gaussian distributed; h as $\mathcal{CN}(0,1)$ and n as $\mathcal{CN}(0,N_0)$. Suppose $h = h_r + jh_i$ and $n = n_r + jn_i$. Because of symmetry, without loss of generality we can assume that $x = \sqrt{E_s}$.

$$\begin{aligned} L &= \frac{4\sqrt{E_s}}{N_0}\Re e\{h^* y\} \\ &= \frac{4|h|^2 E_s}{N_0} + \frac{4\sqrt{E_s}}{N_0}\Re e\{h^* n\} \\ &= \frac{4|h|^2 E_s}{N_0} + \frac{4\sqrt{E_s}}{N_0}(h_r n_r + h_i n_i) \\ &= 4\gamma|h|^2 + \frac{4\sqrt{E_s}}{N_0}(h_r n_r + h_i n_i) \end{aligned} \tag{A.2}$$

where L is LLR of x. Suppose $\gamma = E_s/N_0$. Its straightforward using Eq. (A.2) to show that the pdf of L given h is $\mathcal{N}\left(4\gamma|h|^2, 8\gamma|h|^2\right)$. Suppose that $\mu = 4\gamma|h|^2$ and $\sigma^2 = 8\gamma|h|^2$. Clearly we have $\sigma^2 = 2\mu$. We will use this later for simplification.

Suppose $Z := |L|$. The CDF of Z is given by

$$\begin{aligned} F_Z(z) &= Pr(Z \leq z) = Pr(|L| \leq z) = Pr(-z \leq L \leq z) \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-z}^{z} \exp\left(-\frac{(l-\mu)^2}{2\sigma^2}\right) dl \end{aligned} \tag{A.3}$$

The pdf of Z is given by differentiating $F_Z(z)$ as

$$f_Z(z) = \frac{1}{\sqrt{2\pi\sigma^2}}\left(\exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right) + \exp\left(-\frac{(z+\mu)^2}{2\sigma^2}\right)\right) \tag{A.4}$$

Now $E[|L|] = E[Z]$. After some manipulation it can be calculated as

$$E[Z] = \int_0^\infty z f_Z(z) dz = \mu + \sqrt{\frac{2\sigma^2}{\pi}} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) - 2\mu Q\left(\frac{\mu}{\sigma}\right) \tag{A.5}$$

Converting the expression in terms of μ (we had $\sigma^2 = 2\mu$), we have

$$E[|L|] = \mu + \sqrt{\frac{4\mu}{\pi}} \exp\left(-\frac{\mu}{4}\right) - 2\mu Q\left(\sqrt{\frac{\mu}{2}}\right) \tag{A.6}$$

Substituting the value of $\mu = 4\gamma|h|^2$, we have

$$E[|L|] = 4\gamma|h|^2 + 4\sqrt{\frac{\gamma|h|^2}{\pi}} e^{-\gamma|h|^2} - 8\gamma|h|^2 Q\left(\sqrt{2\gamma|h|^2}\right) \tag{A.7}$$

The approximation used for SPNC is

$$E[|LLR|] \approx 4|h|^2\gamma \tag{A.8}$$

Fig.(A.1)shows the comparison between the exact and approximate value of $E[|LLR|]$. The distance between the source and destination is normalized to 1. It is clear from the figure that the approximation is very accurate even for low and medium SNR region.

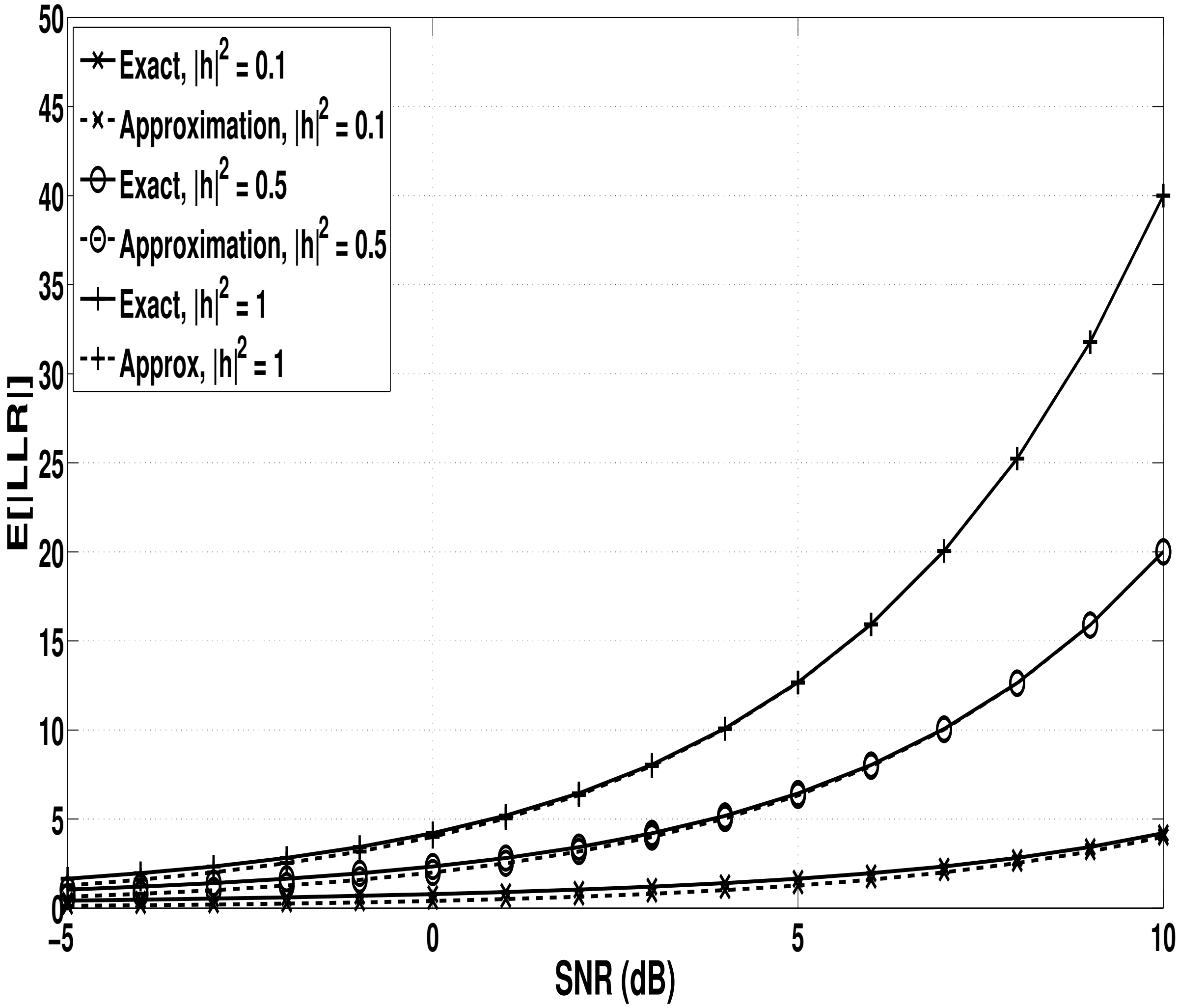

Figure A.1 Comparison of exact Vs approximation of $E[|LLR|]$

APPENDIX B. PROOF OF SPNC FOR INDEPENDENT THRESHOLDS

We will prove the need to have independent thresholds by considering a MARC system with two sources, S_1 and S_2, two relays R_1 and R_2, one destination D. We will prove this by counter-example. We will consider SPNC scheme and another coding scheme with correlated thresholds. We will prove that for the non-prioritized source, the scheme with correlated threshold does not attains maximal diversity order. We refer to the proposed SPNC scheme as first scheme (Eq. 2.13 and 2.14) and second scheme with correlated thresholds as:

$$\mathbf{m}_{r_1} = \begin{cases} \mathbf{m}_1 \oplus \beta_1 \mathbf{m}_2 & \text{if } |h_2|^2 < \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_2|^2 \geq \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\} \end{cases} \tag{B.1}$$

$$\mathbf{m}_{r_2} = \begin{cases} \mathbf{m}_1 \oplus \beta_2 \mathbf{m}_2 & \text{if } |h_2|^2 < \mu \cdot \min\{|h_{r_2}|^2, |h_1|^2\} \\ \mathbf{m}_1 & \text{if } |h_2|^2 \geq \mu \cdot \min\{|h_{r_2}|^2, |h_1|^2\} \end{cases} \tag{B.2}$$

We will prove that for the non-prioritized source, the proposed SPNC scheme achieves maximal diversity order when both the relays R_1 and R_2 transmit $\mathbf{m}_1$. The is the event, denoted as $\mathcal{C}_1$, for which $|h_2|^2 > \mu \ \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 > \mu \ |h_{r_2}|^2$. Correspondingly we will prove that for the non-prioritized source, the second scheme doesn't achieves maximal diversity order when both the relays R_1 and R_2 transmit $\mathbf{m}_1$ which is the event denoted as $\mathcal{C}_1'$, when $|h_2|^2 > \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 > \mu \cdot \min\{|h_{r_2}|^2, |h_1|^2\}$.

First we will consider the outage probability for proposed SPNC scheme for event $\mathcal{C}_1$. The destination decodes message packet of S_1 using maximal ratio combining of direct source-destination transmissions - $\mathbf{y}_1$ and packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$. The message of S_2 is decoded using signal received directly from the source, $\mathbf{y}_2$. The outage probability of non-prioritized source S_2 under the event $\mathcal{C}_1$ is given by

$$P_{o,2}(\mathcal{C}_1) = Pr\left(\{|h_2|^2 < \Gamma\} \cap \{|h_2|^2 > \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\}\} \cap \{|h_2|^2 > \mu \cdot |h_{r_2}|^2\}\right) \tag{B.3}$$

where

$$\Gamma = \frac{2^{4R/2} - 1}{E_s/N_0} \tag{B.4}$$

Denoting $G_1 \triangleq |h_1|^2$, $G_2 \triangleq |h_2|^2$, $G_{r_1} \triangleq |h_{r_1}|^2$ and $G_{r_2} \triangleq |h_{r_2}|^2$, we have

$$P_{o,2}(\mathcal{C}_1) = Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \max\{G_{r_2}, \min\{G_{r_1}, G_1\}\}\}\right) \tag{B.5}$$

The pdf of $G_{1r_1} \triangleq \min\{G_{r_1}, G_1\}$ can be derived as $f(g_{1r_1}) = (\lambda_1 + \lambda_{r_1})e^{-(\lambda_1+\lambda_{r_1})g_{1r_1}}$ [31] where $\lambda_1 = d_1^{\alpha}$, $\lambda_{r_1} = d_{r_1}^{\alpha}$ and . The Eq. (B.5) can be expressed as

$$P_{o,2}(\mathcal{C}_1) = Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \max\{G_{r_2}, G_{1r_1}\}\}\right) \tag{B.6}$$

The pdf of $G_{1r_1r_2} \triangleq \mu \max\{G_{r_2}, G_{1r_1}\}$ can be derived as [31]

$$\begin{aligned} f(g_{1r_1r_2}) = \frac{(\lambda_1 + \lambda_{r_1})}{\mu} e^{\left(-g_{1r_1r_2}\frac{(\lambda_1+\lambda_{r_1})}{\mu}\right)} &+ \frac{\lambda_{r_2}}{\mu} e^{\left(-g_{1r_1r_2}\frac{\lambda_{r_2}}{\mu}\right)} \\ &- \frac{(\lambda_1 + \lambda_{r_1} + \lambda_{r_2})}{\mu} e^{\left(-g_{1r_1r_2}\frac{(\lambda_1+\lambda_{r_1}+\lambda_{r_2})}{\mu}\right)} \end{aligned} \tag{B.7}$$

where $\lambda_{r_2} = d_{r_2}^{\alpha}$. The eq.(B.6) can be written as

$$P_{o,2}(\mathcal{C}_1) = Pr\left(G_{1r_1r_2} < G_2 < \Gamma\right) \tag{B.8}$$

Clearly $G_{1r_1r_2}$ and G_2 are independent. Hence their joint pdf is product of the marginals given by

$$f(g_{12r_1r_2}) = f(g_{1r_1r_2})f(g_2) \tag{B.9}$$

where $f(g_2) = \lambda_2 e^{-\lambda_2 g_2}$, $\lambda_2 = d_2^{\alpha}$. The probability $P_{o,2}(\mathcal{C}_1)$ can be computed by integrating $f(g_{12r_1r_2})$ over appropriate volumes defined by eq.(B.8). After integration the $P_{o,2}(\mathcal{C}_1)$ can be derived as

$$\begin{aligned} P_{o,2}(\mathcal{C}_1) = 1 - e^{-\Gamma\lambda_2} + \frac{\mu\lambda_2\left(e^{-\frac{\Gamma(\lambda_2\mu+\lambda_1+\lambda_{r_1})}{\mu}} - 1\right)}{\lambda_2\mu + \lambda_1 + \lambda_{r_1}} - \frac{\mu\lambda_2\left(e^{-\frac{\Gamma(\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2})}{\mu}} - 1\right)}{\lambda_2\mu + \lambda_1 + \lambda_{r_1} + \lambda_{r_2}} \\ + \frac{\mu\lambda_2\left(e^{-\frac{\Gamma(\lambda_2\mu+\lambda_{r_2})}{\mu}} - 1\right)}{\lambda_2\mu + \lambda_{r_2}} \end{aligned} \tag{B.10}$$

At high SNR, $E_s/N_0 \gg 1$, the third order approximation $e^{-x} \approx 1 - x + x^2/2 - x^3/6$ yields the following asymptotic approximation for $P_{o,2}(\mathcal{C}_1)$

$$P_{o,2}(\mathcal{C}_1) \approx \frac{\Gamma^3 \lambda_2 \left(\lambda_1 + \lambda_{r_1}\right) \lambda_{r_2}}{3\mu^2} \tag{B.11}$$

Clearly the diversity order of $P_{o,2}(\mathcal{C}_1)$ is 3.

Now we will derive the outage probability of second scheme for event denoted as $\mathcal{C}_1'$, which $|h_2|^2 > \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\}$ and $|h_2|^2 > \mu \cdot \min\{|h_{r_2}|^2, |h_1|^2\}$. In this case, both the relays R_1 and R_2 transmit $\mathbf{m}_1$. The destination decodes message packet of S_1 using maximal ratio combining of direct source-destination transmissions - $\mathbf{y}_1$ and packets from relays, $\mathbf{y}_{r_1}$, $\mathbf{y}_{r_2}$. The message of S_2 is decoded using signal received directly from the source, $\mathbf{y}_2$. The outage probability of non-prioritized source S_2 under the event $\mathcal{C}_1'$ is given by

$$P_{o,2}(\mathcal{C}_1') = Pr\left(\left\{|h_2|^2 < \Gamma\right\} \cap \{|h_2|^2 > \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\}\} \cap \{|h_2|^2 > \mu \cdot \min\{|h_{r_1}|^2, |h_1|^2\}\}\right) \tag{B.12}$$

where

$$\Gamma = \frac{2^{4R/2} - 1}{E_s/N_0} \tag{B.13}$$

Denoting $G_1 \triangleq |h_1|^2$, $G_2 \triangleq |h_2|^2$, $G_{r_1} \triangleq |h_{r_1}|^2$ and $G_{r_2} \triangleq |h_{r_2}|^2$, we have

$$P_{o,2}(\mathcal{C}_1') = Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\}\right) \tag{B.14}$$

To calculate $P_{o,2}(\mathcal{C}_1')$, following disjoint events can be considered

Event	Channel Condition	Event	Channel Condition
$\mathcal{E}_1$	$G_1 > G_{r_2} > G_{r_1}$	$\mathcal{E}_2$	$G_1 > G_{r_1} > G_{r_2}$
$\mathcal{E}_3$	$G_{r_2} > G_1 > G_{r_1}$	$\mathcal{E}_4$	$G_{r_2} > G_{r_1} > G_1$
$\mathcal{E}_5$	$G_{r_1} > G_{r_2} > G_1$	$\mathcal{E}_6$	$G_{r_1} > G_1 > G_{r_2}$

Table B.1 Disjoint events for outage probability for second scheme

$$
\begin{aligned}
P_{o,2}(\mathcal{C}_1') = & Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_1\right) \\
& + Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_2\right) \\
& + Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_3\right) \\
& + Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_4\right) \\
& + Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_5\right) \\
& + Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_6\right)
\end{aligned} \tag{B.15}
$$

To prove that the second scheme achieves less than maximal diversity order, consider the events $\mathcal{E}_4$, $\mathcal{E}_5$. The outage probability of S_2 for these events can be stated as

$$
\begin{aligned}
& Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_4\right) \\
& + Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot \min\{G_{r_1}, G_1\}\} \cap \{G_2 > \mu \cdot \min\{G_{r_2}, G_1\}\} \cap \mathcal{E}_5\right) \\
& = Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot G_1\} \cap \{\min\{G_{r_1}, G_{r_2}\} > G_1\}\right)
\end{aligned} \tag{B.16}
$$

Denoting $G_{r_1 r_2} \triangleq \min\{G_{r_1}, G_{r_2}\}$, the pdf of $G_{r_1 r_2}$ can be derived as $f(g_{r_1 r_2}) = (\lambda_{r_1} + \lambda_{r_2})$ $e^{-(\lambda_{r_1}+\lambda_{r_2}) g_{r_1 r_2}}$. Hence the above equation can be stated as

$$
\begin{aligned}
Pr\left(\{G_2 < \Gamma\} \cap \{G_2 > \mu \cdot G_1\} \cap \{G_{r_1 r_2} > G_1\}\right) = & Pr\left(\{\mu \cdot G_1 < G_2 < \Gamma\}\right. \\
& \left. \cap \{\mu \cdot G_{r_1 r_2} > \mu \cdot G_1\}\right)
\end{aligned} \tag{B.17}
$$

The above equation can be further simplified as

$$
\begin{aligned}
Pr\left(\{\mu \cdot G_1 < G_2 < \Gamma\} \cap \{\mu \cdot G_{r_1 r_2} > \mu \cdot G_1\}\right) = & Pr\left(\{\mu \cdot G_1 < \mu \cdot G_{r_1 r_2} < G_2 < \Gamma\}\right) \\
& + Pr\left(\{\mu \cdot G_1 < G_2 < \mu \cdot G_{r_1 r_2} < \Gamma\}\right) \\
& + Pr\left(\{\mu \cdot G_1 < G_2 < \Gamma < \mu \cdot G_{r_1 r_2}\}\right)
\end{aligned} \tag{B.18}
$$

Since $\mu \cdot G_1$, G_2 and $\mu \cdot G_{r_1 r_2}$ are independent random variables, their joint pdf is product of marginals which can expressed as

$$
f(g_{12 r_1 r_2}) = \frac{\lambda_1 \lambda_2 (\lambda_{r_1} + \lambda_{r_2})}{\mu^2} e^{-\left(\frac{\lambda_1}{\mu} g_1 + \frac{\lambda_{r_1}+\lambda_{r_2}}{\mu} g_{r_1 r_2} + \lambda_2 g_2\right)} \tag{B.19}
$$

By integrating $f(g_{12r_1r_2})$ on appropriate volumes defined by Eq.(B.18), the outage probability of second scheme under events $\mathcal{C}_1' \cap \mathcal{E}_4$ and $\mathcal{C}_1' \cap \mathcal{E}_5$ can be calculated as

$$\begin{aligned} &Pr\left(\{\mu \cdot G_1 < \mu \cdot G_{r_1r_2} < G_2 < \Gamma\}\right) \\ &\quad= \mu\left(1 - e^{-\Gamma\lambda_2}\right) - \frac{\left(\lambda_2\mu^2\right)\left(1 - e^{-\frac{\Gamma(\lambda_2\mu+\lambda_{r_1}+\lambda_{r_2})}{\mu}}\right)}{\lambda_2\mu + \lambda_{r_1} + \lambda_{r_2}} - \frac{\left(1 - e^{-\Gamma\lambda_2}\right)\left(\mu\left(\lambda_{r_1} + \lambda_{r_2}\right)\right)}{\lambda_1 + \lambda_{r_1} + \lambda_{r_2}} \\ &\qquad+ \frac{\left(\lambda_2\mu^2\left(\lambda_{r_1} + \lambda_{r_2}\right)\right)\left(1 - e^{-\frac{\Gamma(\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2})}{\mu}}\right)}{\left(\lambda_1 + \lambda_{r_1} + \lambda_{r_2}\right)\left(\lambda_2\mu + \lambda_1 + \lambda_{r_1} + \lambda_{r_2}\right)} \end{aligned} \tag{B.20}$$

At high SNR, $E_s/N_0 \gg 1$, the third order approximation $e^{-x} \approx 1 - x + x^2/2 - x^3/6$ yields the following asymptotic approximation for above probability

$$Pr\left(\{\mu \cdot G_1 < \mu \cdot G_{r_1r_2} < G_2 < \Gamma\}\right) \approx \frac{\Gamma^3\lambda_1\lambda_2\left(\lambda_{r_1} + \lambda_{r_2}\right)}{6\mu} \tag{B.21}$$

Similarly the second term of Eq.(B.18) can be calculated as

$$\begin{aligned} Pr\left(\{\mu \cdot G_1 < G_2 < \mu \cdot G_{r_1r_2} < \Gamma\}\right) &= \frac{\left(\lambda_2\mu^2\left(\lambda_{r_1} + \lambda_{r_2}\right)\right)\left(1 - e^{-\frac{\Gamma(\lambda_2\mu+\lambda_1+\lambda_{r_1}+\lambda_{r_2})}{\mu}}\right)}{\left(\lambda_2\mu + \lambda_1\right)\left(\lambda_2\mu + \lambda_1 + \lambda_{r_1} + \lambda_{r_2}\right)} \\ &\quad+ \frac{\mu\lambda_1\left(1 - e^{-\frac{\Gamma(\lambda_{r_1}+\lambda_{r_2})}{\mu}}\right)}{\lambda_2\mu + \lambda_1} - \frac{\left(\mu\left(\lambda_{r_1} + \lambda_{r_2}\right)\right)\left(1 - e^{-\frac{\Gamma(\lambda_2\mu+\lambda_{r_1}+\lambda_{r_2})}{\mu}}\right)}{\lambda_2\mu + \lambda_{r_1} + \lambda_{r_2}} \end{aligned} \tag{B.22}$$

At high SNR, $E_s/N_0 \gg 1$, the third order approximation $e^{-x} \approx 1 - x + x^2/2 - x^3/6$ yields the following asymptotic approximation for above probability

$$Pr\left(\{\mu \cdot G_1 < G_2 < \mu \cdot G_{r_1r_2} < \Gamma\}\right) \approx \frac{\Gamma^3\lambda_1\lambda_2\left(\lambda_{r_1} + \lambda_{r_2}\right)}{6\mu} \tag{B.23}$$

The third term of Eq.(B.18) can be calculated as

$$\begin{aligned} Pr\left(\{\mu \cdot G_1 < G_2 < \Gamma < \mu \cdot G_{r_1r_2}\}\right) =& \frac{\mu^2\lambda_2}{\lambda_1 + \mu\lambda_2} e^{-\Gamma\frac{(\lambda_1+\mu\lambda_2+\lambda_{r_1}+\lambda_{r_2})}{\mu}} - \mu e^{-\Gamma\frac{(\mu\lambda_2+\lambda_{r_1}+\lambda_{r_2})}{\mu}} \\ &+ \frac{\lambda_1}{\lambda_1 + \mu\lambda_2} e^{-\Gamma\frac{(\lambda_{r_1}+\lambda_{r_2})}{\mu}} \end{aligned} \tag{B.24}$$

At high SNR, $E_s/N_0 \gg 1$, the third order approximation $e^{-x} \approx 1 - x + x^2/2 - x^3/6$ yields the following asymptotic approximation for above probability

$$Pr\left(\{\mu \cdot G_1 < G_2 < \Gamma < \mu \cdot G_{r_1r_2}\}\right) \approx \frac{1}{2}\Gamma^2\lambda_1\lambda_2 - \frac{\Gamma^3\left(2\lambda_2^2\lambda_1\mu + \lambda_1^2\lambda_2 + 3\lambda_2\lambda_1\lambda_{r_1} + 3\lambda_2\lambda_1\lambda_{r_2}\right)}{6\mu} \tag{B.25}$$

Clearly the above term has diversity order of 2 which is less than the maximal achievable diversity order of 3 achievable by the system. Because of this term the overall diversity order of second scheme given by Eq.(B.1) and (B.2) is less than maximal achievable diversity order. Therefore, the second scheme is not a suitable adaptive scheme in which the comparison thresholds at different relays are correlated with each other.

BIBLIOGRAPHY

[1] D. Tse and P. Viswanath, *Fundamentals of wireless communication*, Cambridge University Press, 2005.

[2] E. C. van der Meulen, "Three-terminal communication channels", *Adv. Appl. Probab.*, vol. 3, pp. 120 - 154, 1971.

[3] A. Nosratinia, T. E. Hunter and A. Hedayat, "Cooperative Communication in Wireless Networks", *IEEE Communications Magazine*, pp. 74 - 80, Oct. 2004.

[4] A. Sendonaris, E. Erkip and B. Aazhang, "User Cooperation diversity - Part I: System description", *IEEE Trans. on Communications*, vol. 51, pp. 1927 - 1938, Nov. 2003.

[5] A. Sendonaris, E. Erkip and B. Aazhang, "User Cooperation diversity - Part II: Implementation aspects and performance analysis", *IEEE Trans. on Communications*, vol. 51, pp. 1939 - 1948, Nov. 2003.

[6] T. M. Cover and A. A. E. Gamal,"Capacity Theorems for the Relay Channel", *IEEE Trans. Information Theory*, vol.25, No.5, pp. 572 - 584, Sept. 1979.

[7] J. N. Laneman, D. N. C. Tse and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior", *IEEE Trans. Information Theory*, vol.50, No.12, pp. 3062 - 3080, Dec. 2004.

[8] R. Ahlswede, N. Cai, S. Y. R Li and R. W. Yeung, "Network Information Flow", *IEEE Trans. on Information Theory*, Vol. 46, pp. 1204 - 1216, July 2000.

[9] A. Stefanov and E. Erkip," Cooperative Coding for Wireless Networks", *IEEE Trans. on Communications*, Vol. 52, No. 9, pp. 1470 - 1476, Sep. 2004.

[10] T. Hunter and A. Nosratinia, " Diversity Through Coded Cooperation", *IEEE Trans. on Wireless Communications*, Vol. 5, No.2, pp. 283 - 289, Feb. 2006.

[11] Y. Lin, B. Li and B. Liang, " Differentiated Data Persistence with Priority Random Linear Codes", in *Proc. ICDCS*, pp. 47, 2007.

[12] D. Silva and F. R. Kschischang, " Rank Metric Codes for Priority Encoding Transmission with Network Coding", *Canadian Workshop on Information Theory*, pp. 81 - 84, 2007.

[13] A. Limmanee and W. Henkel, " UEP Network Coding for Scalable Data", in *Proc. 5th International Symposium on Turbo Codes and Related Topics*, 2008, pp. 333 - 337.

[14] N. Thomos, J. Chakareski and P. Frossard, "Randomized Network Coding for UEP Video Delivery in Overlay Networks", in *Proc. ICME*, 2009.

[15] M.K.Simon and M.S.Alouini,*Digital Communication Over Fading Channels*, 2nd Edition, Wiley Interscience, 2005.

[16] J. Craig, "New, simple and exact result for calculating the probability of error for two dimensional signal constellations", *Proc. Military Communication Conference*, pp.25.5.1 - 25.5.5, November 1991.

[17] K.Yu, J. Evans and I.Collings , "Performance Analysis of Pilot Symbol Aided QAM for Rayleigh Fading Channels", in *Proc. of IEEE Conference on Communications (ICC '02)*, vol. 3, pp. 1731 - 1735, August 2002.

[18] S. Han, S. Ahn, E. Oh and D. Hong, "Effect of Channel Estimation Error on BER Performance in Cooperative Transmission", *IEEE Trans. Veh. Technol.*, vol. 58, No. 4, pp. 2083 2088, May 2009.

[19] M. Iezzi, M. Di Renzo and F. Graziosi, "Network Code Design from Unequal Error Protection Coding: Channel - Aware Receiver Design and Diversity Analysis", in *Proc. IEEE Int. Conf. on Communications. (ICC '11)*, Kyoto, Japan, June 2011.

[20] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes", *IEEE Trans. Infor. Theory*, vol.42, no.2, pp.429 - 445, March 1996.

[21] D.H. Woldegebreal and H. Karl, "Multiple - access relay channel with network coding and nonideal source-relay channels", In *Proc. 4th International Symposium on Wireless Communication Systems*, pp. 732 - 736, Oct. 2007.

[22] J. Nocedal and S. J. Wright, *Numerical Optimization*, 2nd ed., New York: Springer, 2006.

[23] M. Xiao and M. Skoglund, "Multiple User Cooperative Communications Based on Linear Network Coding", *IEEE Trans. Commun.*, vol. 58, No.12, Dec. 2010.

[24] A. Sendonaris, E. Erkip and B. Aazhang, "User Cooperation Diversity - Part I: System description", *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927 - 1938, Nov. 2003.

[25] A. Sendonaris, E. Erkip and B. Aazhang, "User Cooperation Diversity - Part II: Implementation aspects and performance analysis", *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939 - 1948, Nov. 2003.

[26] J. N. Laneman and G. W. Wornell, "Distributed Space - Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks", *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415 - 2425, Oct. 2003.

[27] L. Zheng and D. N. C. Tse,"Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels", *IEEE Tran. Info. theory*, Vol. 49, No. 5, pp. 1073 - 1096, May 2003.

[28] H. Topakkaya and Z. Wang, "Wireless network code design and performance analysis using diversity - multiplexing tradeoff", *IEEE Trans. Commun.*, vol. 59, no. 2, pp. 488 - 496, Feb. 2011.

[29] J.L. Rebelatto, B.F. Uchoa Filho, Y. Li and B. Vucetic, "Multiuser Cooperative Diversity Through Network Coding Based on Classical Coding Theory", *IEEE Trans. on Signal Processing*, Vol. 60, No. 2, pp. 916 - 926, Feb. 2012.

[30] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd ed., John Wiley & Sons, Inc., 2003.

[31] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed., Tata McGraw Hill, 2002.

[32] J. Nocedal and S. J. Wright, *Numerical Optimization*, 2nd ed., New York: Springer, 2006.

[33] A. Bletsas, A. Khisti, D. P. Reed and A. Lippman, "A Simple Cooperative Diversity Method Based on Network Path Selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp.659 - 672, March 2006.

[34] A. Bletsas, H. Shin and M.Z. Win, "Cooperative Communications with Outage Optimal Opportunistic Relaying," *IEEE Trans. Wireless Commun.*, vol.6, no.9, Sep. 2007.

[35] C. Hausl and P. Dupraz, "Joint network - channel coding for the multiple - access relay channel," *Proc. of SECON* 2006, vol. 3, pp. 817 - 822, Sept. 2006.

[36] S. Zhang, Y. Zhu, S. Liew and K. Letaief, "Joint design of network coding and channel decoding for wireless networks," in *Proc. of WCNC 2007*, pp. 779 - 784, Mar. 2007.

[37] Z. Guo, J. Huang, B. Wang, S. Zhou, J.H. Cui, P. Willett, "A Practical Joint Network - Channel Coding Scheme for Reliable Communication in Wireless Networks", *IEEE Trans. Wireless Commun.*, vol.11, no.6, pp. 2084 - 2094, June 2012.

[38] C. Peng, Q. Zhang, M. Zhao, Y. Yao and W. Jia, "On the Performance Analysis of Network - Coded Cooperation in Wireless Networks", *IEEE Trans. Wireless Commun.*, vol.7, no.8, pp. 3090 - 3097, Aug. 2008.

[39] J. Manssour, A. Osseiran and S. Ben Slimane, "Wireless Network Coding in Multi - Cell Networks: Analysis and Performance," in *Proc. of Int. Conf. on Sig. Proc. and Commun. Systems ICSPCS 2008*, Australia, Dec. 2008.

[40] J. Manssour, A. Osseiran and S. Ben Slimane, "Opportunistic relay selection for wireless network coding," *IEEE 9th Malaysia International Conference on Communications (MICC)*, 2009.

[41] S. Sharma, S. Yi, Y. T. Hou, H.D. Sherali, S. Kompella, "Optimizing network - coded cooperative communications via joint session grouping and relay node selection," in *Proc. IEEE INFOCOM*, pp. 1898 - 1906, 2011.

[42] S. Sharma, S. Yi, Y. T. Hou, S. Kompella, S.F. Midkiff, "Optimal grouping and matching for network - coded cooperative communications," in *Proc. of Military Communications Conference 2011*, pp. 722 - 728, 2011.

[43] Z. Mobini, P. Sadeghi and S. Zokaei, "Joint power allocation and relay selection in network - coded multi - unicast systems", in *Proc. of WCNC 2012*, pp. 1113 - 1118, April 2012.

[44] A.H.M. Hassan, B. Dai, B. Huang, E. Eisa and M. Azhar, "Relay Selection for Multiple Access Relay Channel with Decode - Forward and Analog Network Coding," *International Journal of Distributed and Parallel Systems (IJDPS)*, Vol.3, No.2, March 2012.

[45] R. Pyndiah, "Near - optimum decoding of product codes: Block turbo codes," *IEEE Tran. on Commun.*, vol. 46, pp. 1003 - 1010, Aug. 1998.

[46] P.A. Martin and D.P. Taylor, "On multilevel codes and iterative multistage decoding", *IEEE Tran. on Commun.*, Vol. 49, pp. 1916 - 1925, 2001.

[47] D. Chase, "A class of algorithms for decoding block codes with channel measurement information," *IEEE Tran. on Info. Theory*, vol. IT - 18, pp. 170 - 182, Jan. 1972.

[48] Y. Chen, S. Kishore, J. Li, "Wireless diversity through network coding," *IEEE Wireless communications and networking conference (WCNC)*, April 2006.

[49] X. Bao, J. Li, "Matching code - on - graph with networks - on - graph: adaptive network coding for wireless relay networks," *Proc. Allerton Conf. on Commun., Control and Computing*, Urbana Champaign, IL, Sept. 2005.

[50] Sang W. Kim, "Cooperative concatenated coding in multi - hop multiple access relay networks," *IEEE Transactions on Communications*, pp. 3305 - 3315, Nov. 2010.

[51] F. Zhao, T. Kalkert, M. Medard, K. J. Han, "Signatures for content distribution with network coding," *Proc. of IEEE International Symposium on Information Theory*, Nice, France, June 2007.

[52] Y. Mao, M. Wu, "Tracing malicious relays in cooperative wireless communications," *IEEE Trans. on Info. Forensics and Security*, vol. 2, no. 2, pp. 198 - 212, June 2007.

[53] S. Dehnie, H. T. Sencar, N. Memon, "Detecting malicious behavior in cooperative diversity," *Proc. of Conf. on Information Sciences and Systems CISS*, 2007.

[54] T. Ho, B. Leong, R Koetter, M. Medard, M. Effros, D. R. Karger, "Byzantine modification detection in multicast networks with random network coding," *IEEE Trans. Info. Theory*, vol. 54, no. 6, pp. 2798 - 2803, June 2008.

[55] S. Jaggi, M. Langberg, S. Katti, T. Ho, D. Katabi, M. Medard, M. Effros, "Resilient network coding in the presence of byzantine adversaries," *IEEE Trans. Info. Theory*, vol. 54, no. 6, pp. 2596 - 2603, June 2008.

[56] Guanfeng Liang, Rachit Agarwal and Nitin Vaidya, "When Watchdog Meets Coding," *Proc. of IEEE INFOCOM*, March 2010.

[57] Sang W. Kim, "Securing wireless network coding against pollution attack at the physical layer," *Proc. of MILCOM*, 2012.

[58] Zhen Yu, Yawen Wei, Bhuvaneswari Ramkumar, and Yong Guan, "An Efficient Signature Scheme for Securing XOR Network Coding against Pollution Attacks," in *Proc. of IEEE INFOCOM*, 2009.

[59] S. Agrawal and D. Boneh, "Homomorphic MACs: MAC - Based Integrity for Network Coding," in *Proc. Applied Cryptography and Network Security*, pp. 292 - 305, 2009.

[60] D. Charles, K. Jain, and K. Lauter, "Signature for Network Coding," in *Proc. of Conf. on Information Sciences and Systems (CISS)*, 2006.

[61] R. U. Nabar, Ö. Oyman, H. Bölcskei, A. J. Paulraj, "Capacity scaling laws in MIMO wireless networks," in *Proc. of Allerton Conf. on Commun. Control and Comp.*, Monticello, USA, Oct. 2003.

[62] A. Wittneben, "A theoretical analysis of multiuser zero forcing relaying with noisy channel state information," in *Proc. IEEE VTC*, May 2005.

[63] C. Esli, S. Berger, A. Wittneben, "Optimizing Zero - Forcing Based Gain Allocation for Wireless Multiuser Networks," in *Proc. IEEE ICC*, Glasgow, Scotland, June 2007.

[64] R. Louie, Y. Li, B. Vucetic , "Zero Forcing Processing in Two Hop Networks with Multiple Source, Relay and Destination Nodes," *Proc. IEEE ICC*, Dresden, Germany, June 2009.

[65] S. Berger, A. Wittneben, "Cooperative distributed multiuser MMSE relaying in wireless ad - hoc networks," in *Proc. IEEE Signals, Systems and Computers Conf.*, November 2005.

[66] R. Krishna, Z. Xiong, S. Lambotharan, "A cooperative MMSE relay strategy for wireless sensor networks," *IEEE Signal Process. Letters*, vol. 15, pp. 549 552, 2008.

[67] N. Khajehnouri, A. H. Sayed, "Distributed MMSE relay strategies for wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 55, pp. 3336 3348, July 2007.

[68] H. Shi, T. Abe, T. Asai, H. Yoshino, "Relaying schemes using matrix triangularization for MIMO wireless networks," *IEEE Trans. Commun.*, vol.55, no.9, pp. 1683 1688, Sep.2007.

[69] Y. Jing, H. Jafarkhani, "Network beamforming using relays with perfect channel information," in *Proc. IEEE ICASSP*, April 2007.

[70] D. H. N. Nguyen, H. H. Nguyen, H. D. Tuan, "Distributed Beamforming in Relay - Assisted Multiuser Communications," in *Proc. IEEE ICC*, Dresden, Germany, June 2009.

[71] Salama S. Ikki, Mohamed H. Ahmed, "On the Performance of Cooperative - Diversity Networks with the N^{th} Best - Relay Selection Scheme," *IEEE Trans. on Commun.*, Vol. 58, No. 11, pp. 3062 - 3069, Nov. 2010.

[72] E. Biglieri, *Coding for Wireless Channels*, Springer, 2005.

[73] E. K. Hall, S. G. Wilson, "Design and Analysis of Turbo Codes on Rayleigh Fading Channels", *IEEE J. Sel. Areas Commun.*, Vol. 16, No. 2, pp.160 - 174, Feb. 1998.

[74] S. Lin, D. J. Costello, *Error Control Coding*, 2nd Ed., Prentice Hall, 2004.